High Power Microwave Technology and Effects

A University of Maryland Short Course

Presented to MSIC

Redstone Arsenal, Alabama

August 8-12, 2005

HPM Bibliography

- High Power Microwave Sources,
  eds. Granatstein and Alexeff, Artech House, 1987
- High Power Microwaves,
  by Benford and Swegle, Artech House, 1991
- High Power Microwave Systems and Effects,
  by Taylor and Giri, Taylor and Francis, 1994
- Applications of High Power Microwaves,
  eds. Gaponov- Grekhov & Granatstein, Artech House, 1994
- Generation and Application of High Power Microwaves,
  eds. Cairns and Phelps, J.W. Arrowsmith Ltd., 1977
- High Power Microwave Sources and Technologies,
  eds. Barker and Schamiloglu, IEEE Press, 2001
- High-Power Electromagnetic Radiators, Nonlethal Weapons & Other Applications, by Giri, Harvard Univ. Press, 2004
- Modern Microwave and Millimeter-wave Power Electronics,
  eds. Barker, Booske, Luhman and Nusinovich,
  IEEE Press & Wiley-Interscience, 2005

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A. Kehs, ARL

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K. Hendricks, AFRL

Course Outline

- Aug 8 – Aug. 9, HPM Technology

(Sources, Waveguides, Antennas, Propagation)

Presenter: Victor Granatstein (vlg@umd.edu)

- Aug. 10, Microwave Upset of Electronic Ckts.

Presenter: John Rodgers (rodgers @umd.edu)

- Aug.11, Chaos & Statistics of Microwave Coupling

Presenter: Steve Anlage (anlage@squid.umd.edu)

- Aug. 12, Failure Mechanisms in Electronic Devices

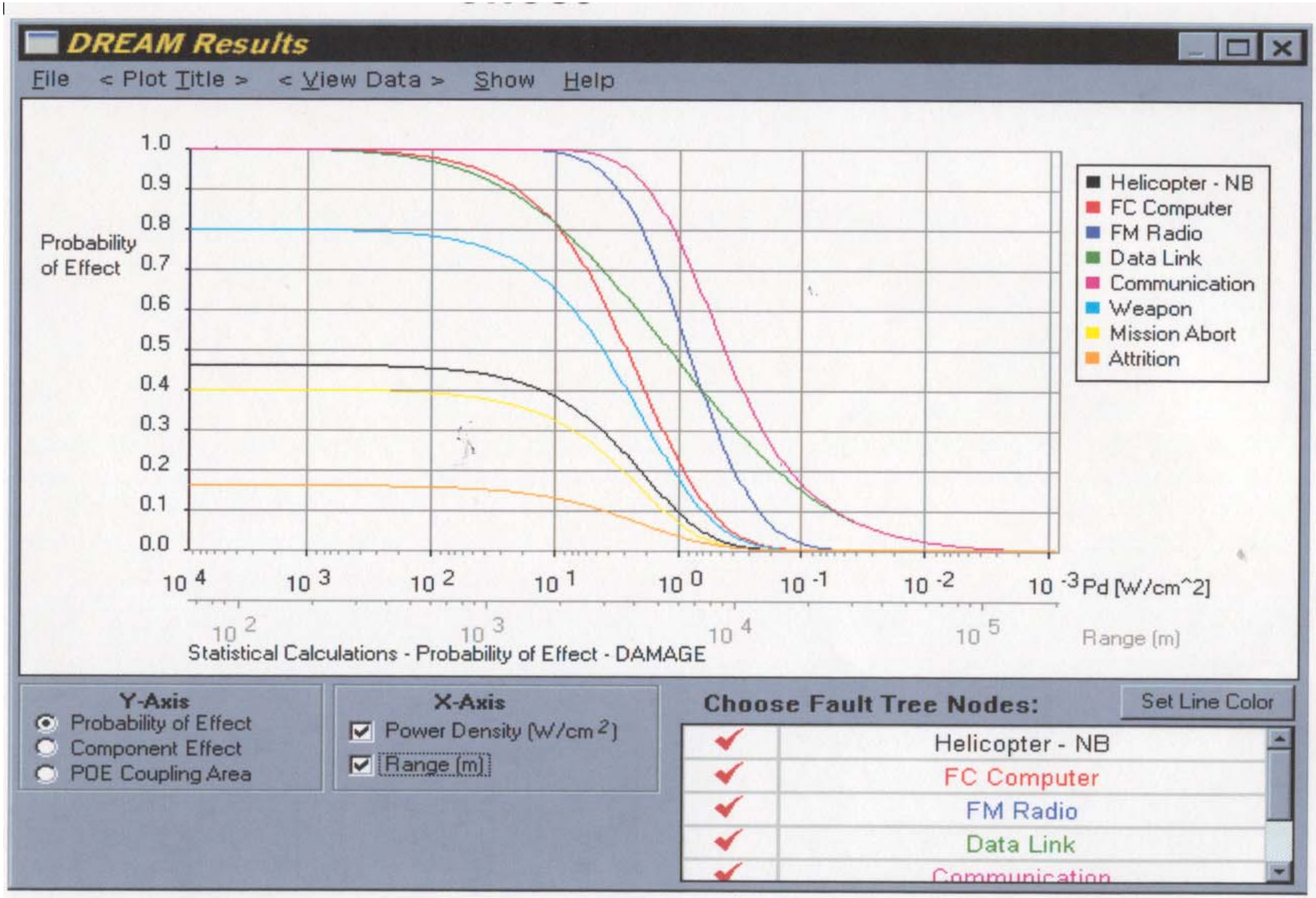
Presenter: Neil Goldsman (neil@eng.umd.edu)

RF-Effects Definitions

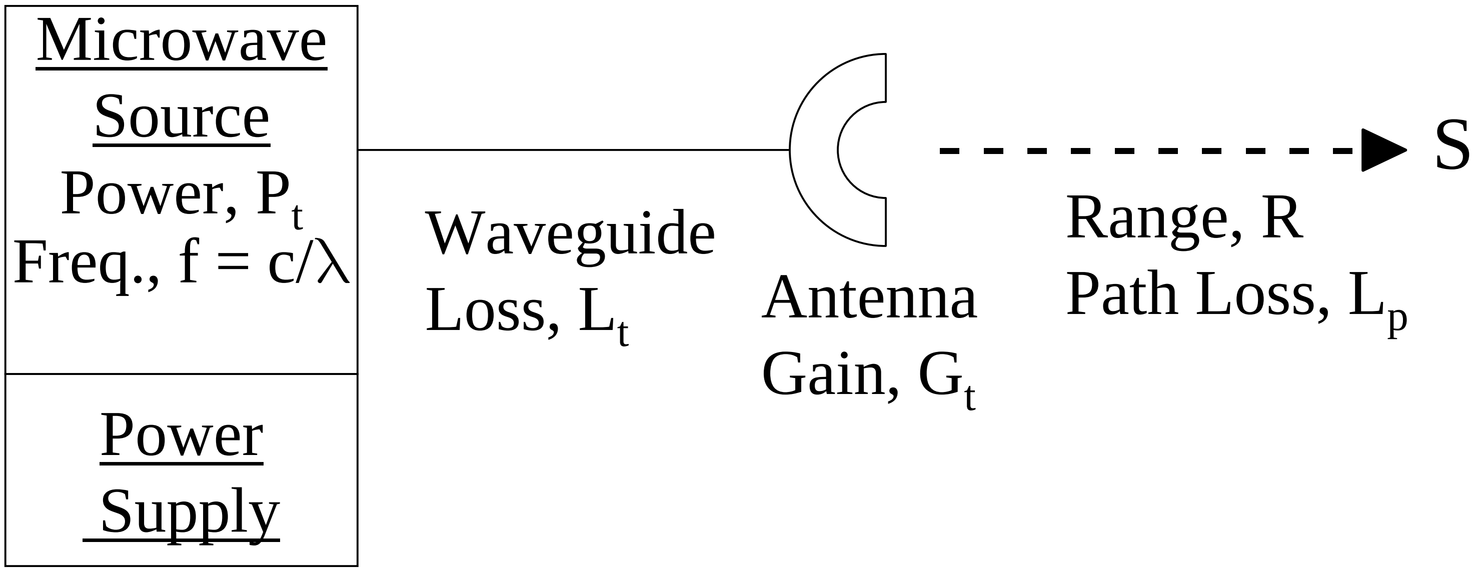
5	Damage	- Requires hardware, software, or firmware replacement
4	Upset	- Requires external intervention
3	Disturbance	- Effect present after illumination but eventually recovers
2	Interference	- Effect present only when illuminated
1	No Effect	
0	Unknown/Not Observed	

Estimate of Required Power Density on "Target"

(using "DREAM" a PC compatible application that estimates probability of RF upset or damage of a system's electronics)



HPM Sources, Waveguides, Antennas and Propagation



Effective Isotropic Radiated Power, EIRP = $P_t \, G_t \, / \, L_t$
Power Density at "Target", $S = (4\pi/\lambda^2)$ (EIRP/ L_p)

In free space, $L_p = L_f = (4\pi R \, / \, \lambda \,)^2$ and $S = \text{EIRP} \, / \, (4\pi \, R^2)$

A Note on Using Decibels

- Decibels are a dimensionless comparison of a power to a reference power : $P_s(dB) = 10 \log [P/ P_{ref}]$
  e.g., P = 1000 Watts
  $P(dBW) = 10 \log [P/(1 \text{ Watt})] = 30$ dBW
  dBW indicates that the reference power is 1 Watt
- P_t, P_r can be expressed in dBW (power compared with 1 W)
- G_t, G_r can be expressed in dBi (enhancement in power density in max. direction compared with isotropic radiator)
- L, L_t, L_r can be expressed in dB

Example: $EIRP = P_t G_t / L_t$

$EIRP(dBW) = P_t (dBW) + G_t (dBi) - L_t (dB)$

If $P_t = 63$ dBW, $G_t = 23$ dBi, $L_t = 2$ dB

Then, EIRP(dBW) = 63 dBW + 23 dBi – 2dB = 84 dBW

or EIRP = $10^{8.4}$ Watts = 2.51×10^8 Watts = 251 Megawatts

HIGH POWER MICROWAVE SOURCES

HPM Weapon Sources

HPM weapon sources are

- Designed to produce electromagnetic interference or damage with:
  - peak radiated power level of 100 MW or more (100 kV/m), or
  - pulsed radiated energy of 1 Joule per pulse.
- Generally categorized as belonging to one of two types:
  - Narrowband: frequencies above 300 MHz and below 300 GHz, usually between 1 GHz and 35 GHz; frequency bandwidth less than 10% of the carrier frequency.
  - Wideband or Ultra-wideband (UWB): frequency bandwidth is greater than 10% of the mean frequency (e.g., system which extends from 10 MHz to few GHz).

Narrowband HPM Sources

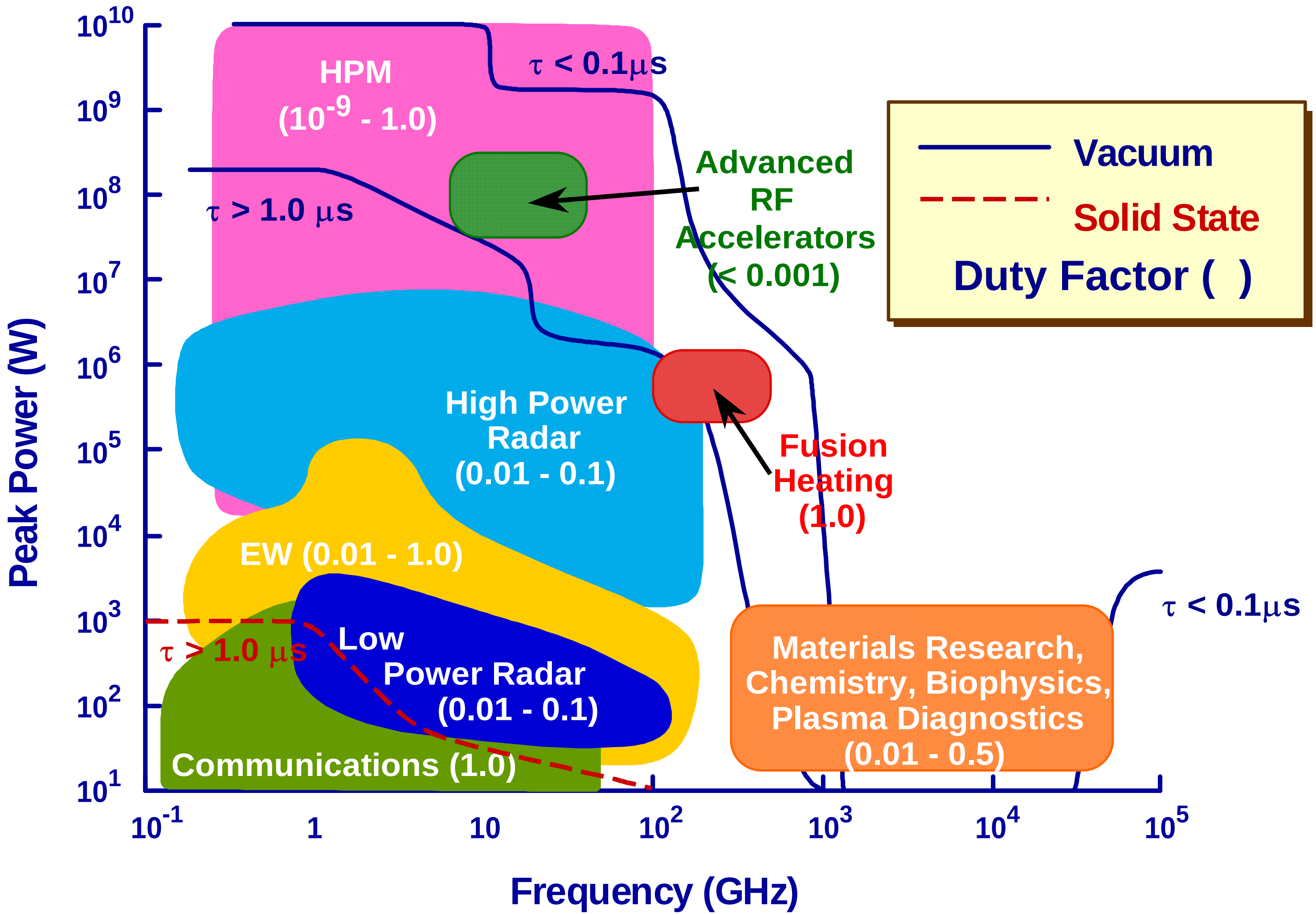
- Strongest effects observed for
  300 MHz $< f <$ 3 GHz (1 meter $> \lambda >$10 cm)
- Pulsed modulation is more effective than CW
  with pulse duration 100 nsec $< \tau <$ 1 μsec
- $P_{av} = P_P \times \tau \times PRF$

  PRF is Pulse Repetition Frequency

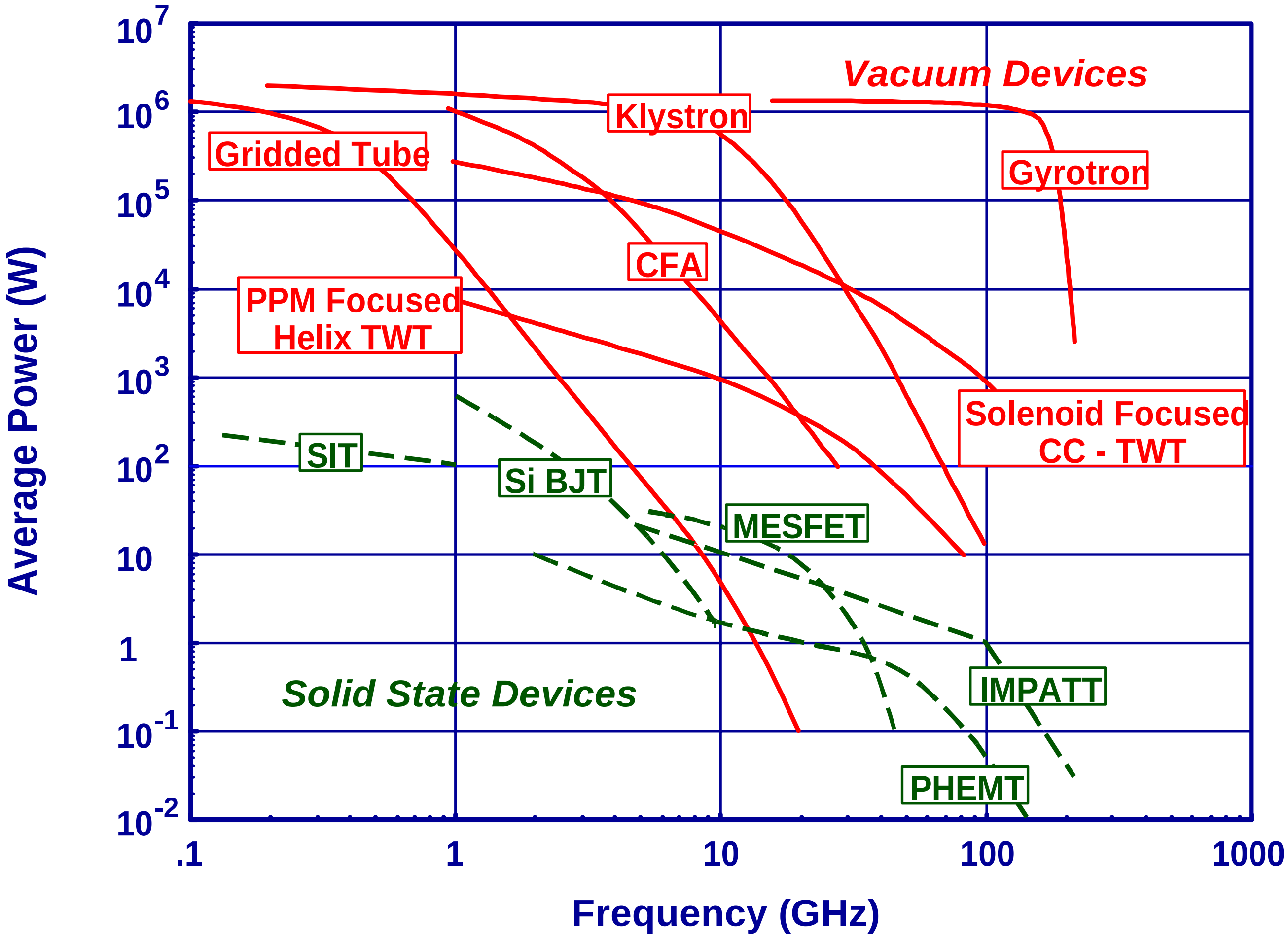
The Radio Frequency Spectrum

ELF	Extremely Low Frequency	< 3 kHz	RADIO WAVES
VLF	Very Low Frequency	3 - 30 kHz	
LF	Low Frequency	30 - 300 kHz	
MF	Medium Frequency (AM radio, “ground wave”)	300 - 3000 kHz	
HF	High Frequency (shortwave radio , “sky wave”)	3 - 30 MHz	
VHF	Very High Frequency (FM radio, TV)	30 - 300 MHz	
UHF	Ultra High Frequency (UHF-TV, mobile phones, GPS)	300MHz - 3GHz	<-------- (boundary)
SHF	Super High Frequency (Radar)	3 - 30 GHz	MICRO-WAVES
EHF	Extremely High Frequency (millimeter-waves)	30 - 300 GHz	

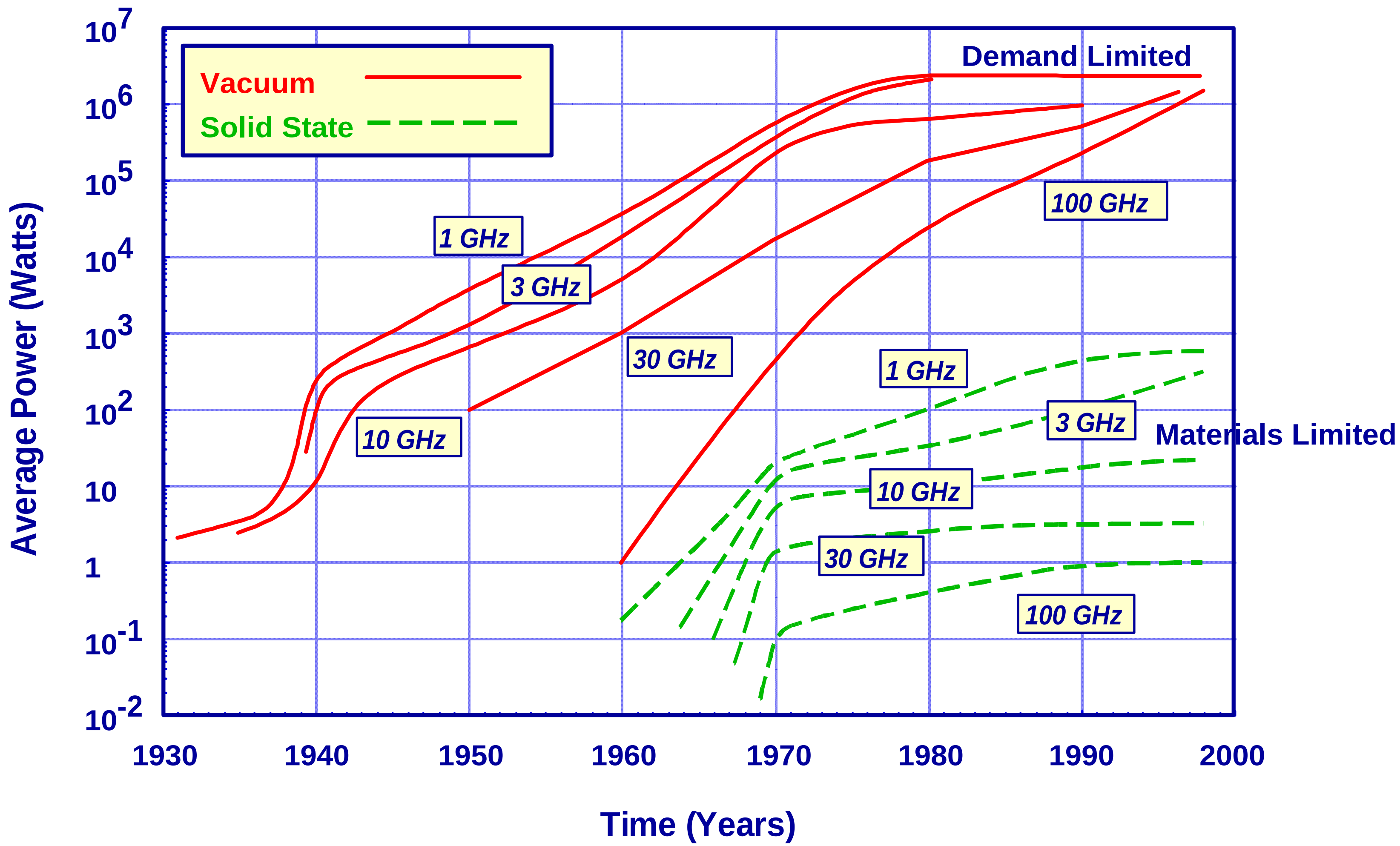
Domains of Application: Single Device Peak Power Performance Limits



Source Performance (Average Power) State of Technology



Microwave Source Technology
Growth Rate of Average Power



Physics of HPM sources

Physics of HPM sources is very much the same as the physics of traditional microwave vacuum electron devices. However,

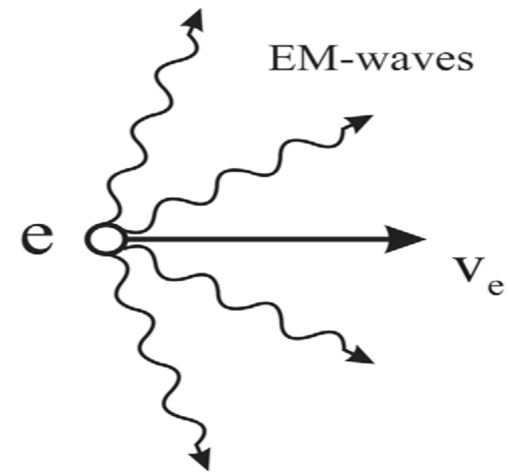
a) Some new mechanisms of microwave radiation are possible (e.g., cyclotron maser instability);

b) Some peculiarities in the physics of wave-beam interaction occur at high voltages, when electron velocity approaches the speed of light.

History of HPM sources starts from the late 1960's when the first high-current accelerators (V>1 MeV, I>1 kA) were developed (Link, 1967; Graybill and Nablo, 1967; Ford et al, 1967)

Microwave radiation by “free” electrons

In practically all sources of HPM radiation, the radiation is produced by electrons propagating in the vacuum (free electrons).



How to force electrons to radiate electromagnetic (EM) waves?

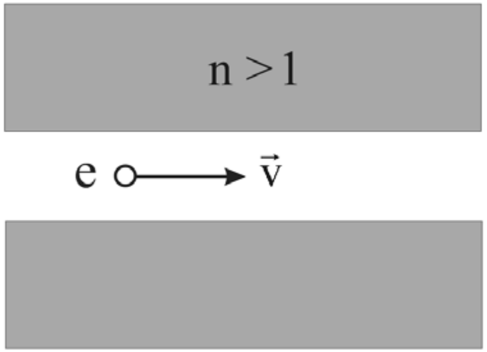
An electron moving with a constant velocity in vacuum does not radiate!

Hence, electrons should either move with a variable velocity or with a constant velocity, but not in vacuum.

Three kinds of microwave radiation

I. Cherenkov radiation:

Electrons move in a medium where their velocity exceeds the phase velocity of the EM wave.

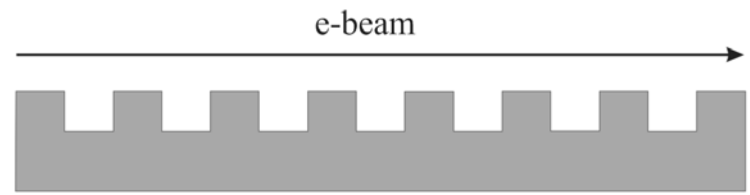


When the medium can be characterized by a refractive index, *n*, the wave phase velocity, v_{ph}, there is equal to *c/n* (where *c* is the speed of light). So, in media with *n>1*, the waves propagate slowly (slow waves) and , hence, electrons can move faster than the wave:

$$v_{ph} = c/n < v_{el} < c$$

In such a case the electrons can be decelerated by the wave, which means that electrons will transfer a part of their energy to the wave. In other words, the energy of electrons can be partly transformed into the energy of microwave radiation.

IA. Smith-Purcell radiation



When there is a periodic structure (with a period *d*) confining EM waves, the fields of these waves can be treated as superposition of space harmonics of the waves (Floquet theorem)

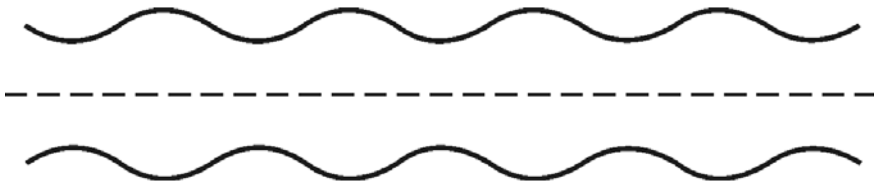
$$\vec{E} = \mathrm{Re}\left\{ A \sum_{l=-\infty}^{\infty} \alpha_l e^{i(\omega t - k_l z)} \right\}$$

where $k_l = k_0 + l2\pi / d$ **is the wave propagation constant for the *l*-th space harmonic. Thus, the wave phase velocity of such harmonics *(l>1)*,** $v_{ph,l} = \omega / k_l$ **, can be smaller than the speed of light and therefore for them the condition for Cherenkov radiation can be fulfilled.**

Smith-Purcell radiation can be treated as a kind of Cherenkov radiation.

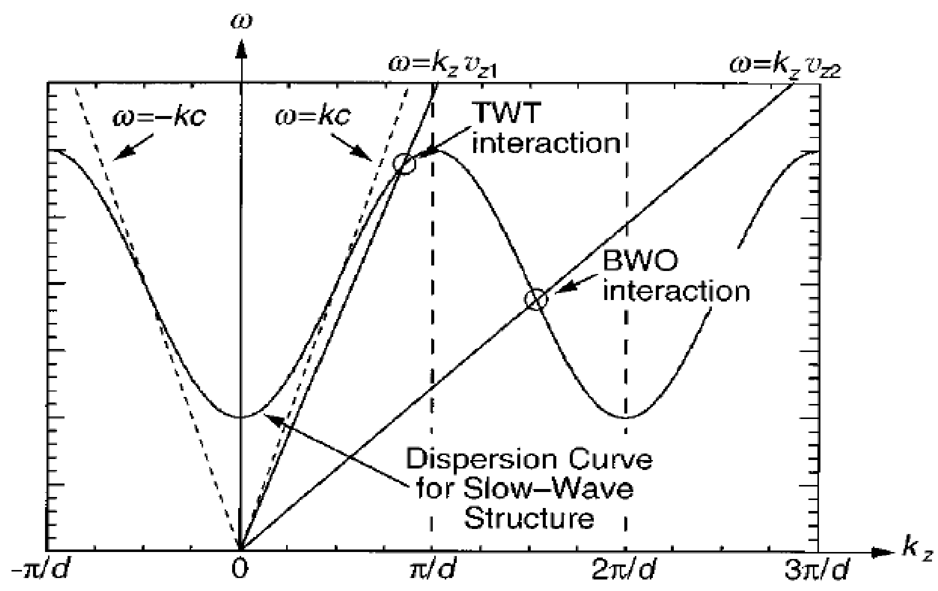
Slow-wave structures (cont.)

C. Rippled-wall SWS



The wave group velocity can be either positive (TWT) or negative (BWO)

$$v_{gr} = \partial\omega / \partial k_z$$

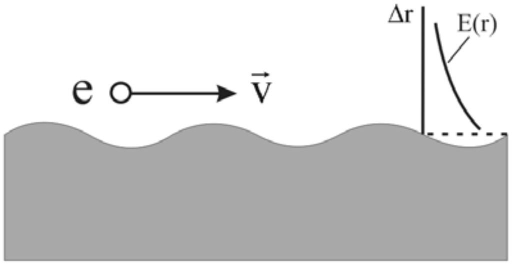


Group velocity is the speed of propagation of EM energy along the Waveguide axis

Slow waves

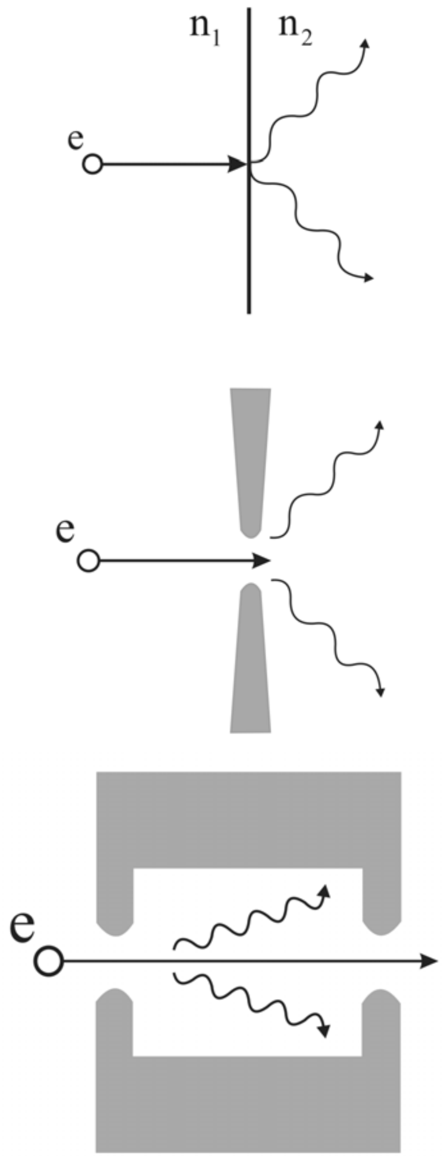
When the wave propagates along the device axis with phase velocity ω/k_z smaller than the speed of light c this means that its transverse wavenumber $k_\perp$ has an imaginary value, because $k_\perp^2 = (\omega / c)^2 - k_z^2 < 1$

This fact means localization of a slow wave near the surface of a slow-wave structure. An electron beam should also be located in this region to provide for strong coupling of electrons to the wave.



As the beam voltage increases, the electron velocity approaches the speed of light. Correspondingly, the wave can also propagate with the velocity close to speed of light. (Shallow slow-wave structures)

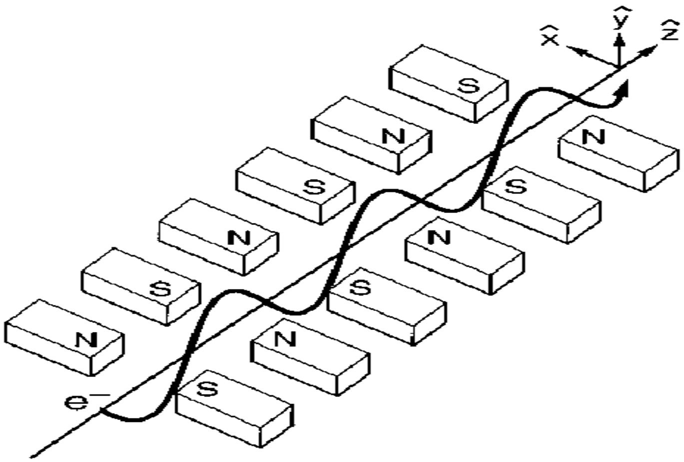
II. Transition radiation



In a classical sense, TR occurs when a charged particle crosses the border between two media with different refractive indices.
The same happens in the presence of some perturbations in the space, such as conducting grids or plates.
(Grids in RF tubes, e.g., triodes etc).
Cavities with small holes for beam transport can play the role of such perturbations as well.

III. Bremsstrahlung

This sort of radiation occurs when electrons exhibit oscillatory motion in external magnetic or electric fields. These fields can be either constant or periodic.



Example: electron motion in a wiggler, which is a periodic set of magnets

Doppler-shifted wave frequency is equal to the frequency of electron oscillations, Ω , or its harmonic:

$$\omega - k_z v_z = s\Omega$$

Coherent radiation

So far, we considered the radiation of a single particle.
This radiation is called spontaneous radiation.
In HPM sources, a huge number of electrons N passes through the interaction space.
For instance, in the case of a 1-MV, 1-kA e-beam about $6 \cdot 10^{12}$ particles pass through the interaction space every nsec.

When these particles radiate electromagnetic waves in phase, i.e. **<u>coherently</u>, the radiated power scales as N^2 while in the case of spontaneous radiation the radiated power is proportional to N**

How to force this huge number of particles to radiate coherently?

Coherent radiation (cont.)

Electrons can radiate in phase when they are gathered in compact bunches.
In some cases such bunches can be prepared in advance (photo-emitters).
Most often, however, the bunches are formed in the interaction space as a result of interaction between the RF field and initially uniformly distributed electrons.

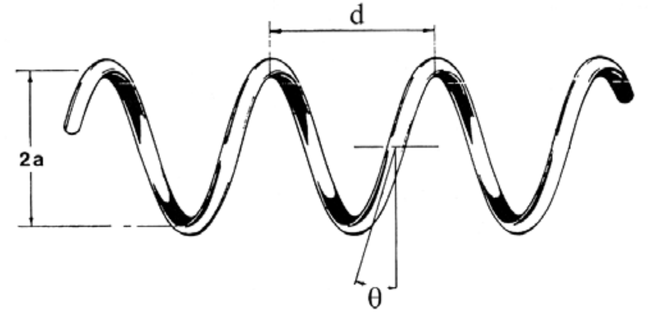
Sources of coherent Cherenkov/ Smith-Purcell radiation

- Traveling-wave tubes
- Backward-wave oscillators
- Magnetrons (cross-field devices)
- MILOs

To provide the synchronism between electrons and EM waves, in all these devices periodic slow-wave structures are used.

Slow-wave structures (SWS's)

A. Helix slow-wave structure



Assume that the wave propagates along the wire with the speed of light

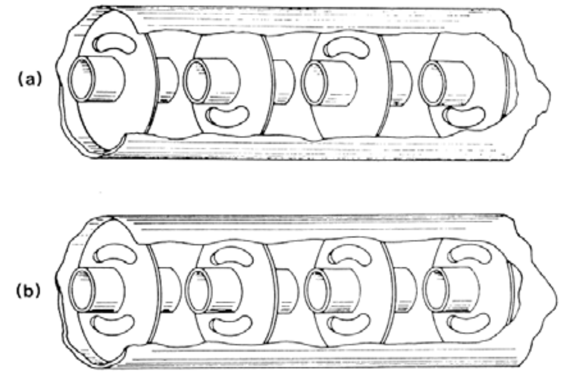
Pitch angle $\tan\theta = 2\pi a / d$

Phase velocity of the wave along the axis $v_{ph} = c\sin\theta$

This phase velocity does not depend on frequency.
No dispersion! $v_{ph}(\omega) = const$
Electrons can be in synchronism with the wave of an arbitrary frequency -very large bandwidth is possible.

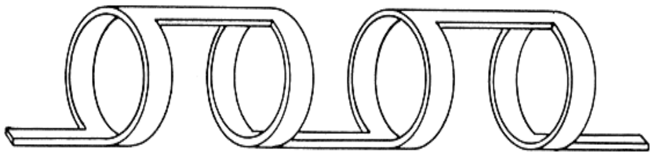
Slow-wave structures (cont.)

B. Coupled-cavity SWS's

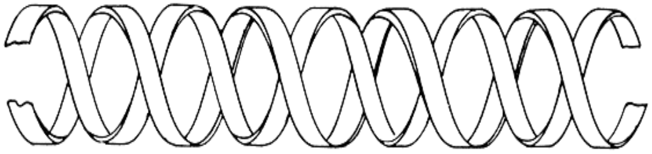


These SWS's do have dispersion.
However, they can handle a higher level of microwave power.
Thus, they can be used in the devices intended for high-power, moderate bandwidth applications.

Slow-wave structures (cont.)



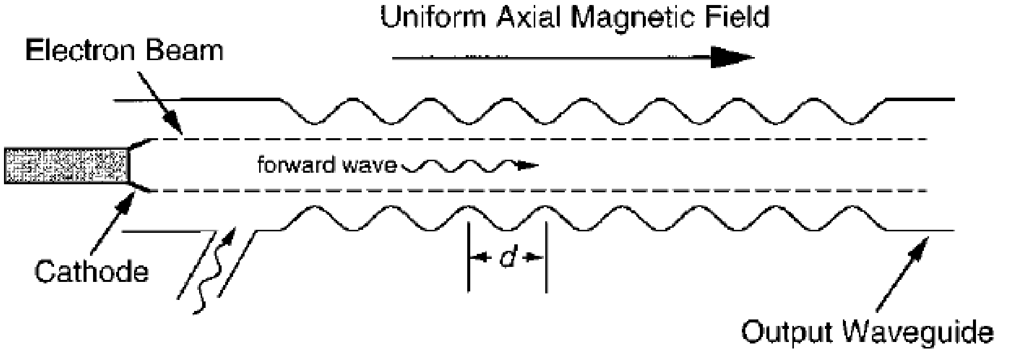
(a) Ring-bar SWS



(b) Bifilar helix SWS

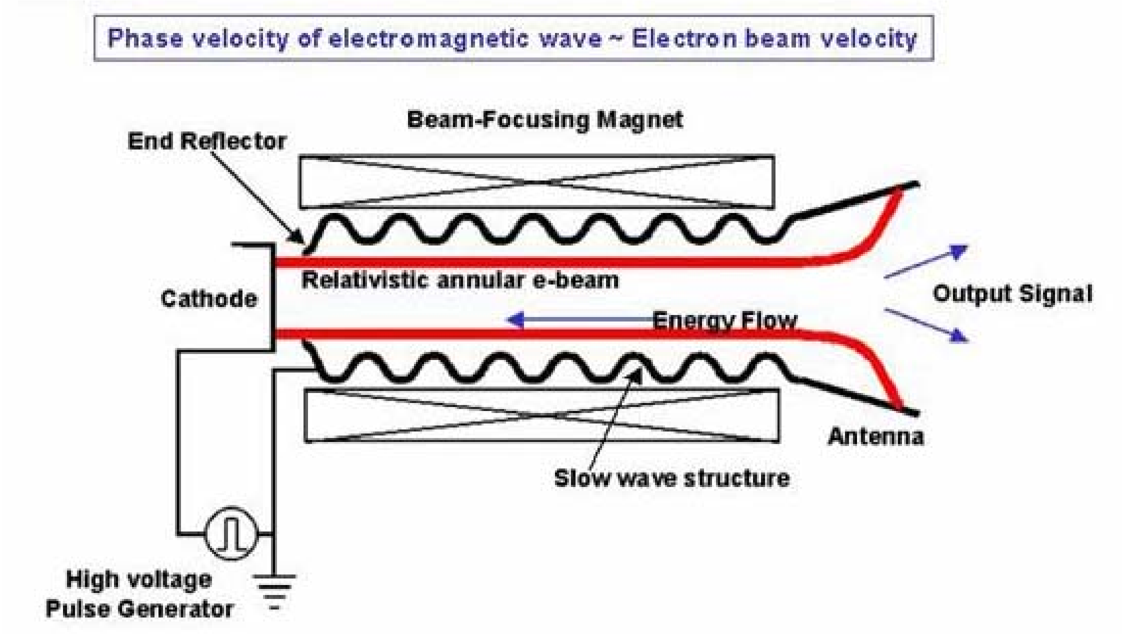
Traveling-wave tubes (TWT's)

Electrons moving linearly with the axial velocity v_{z0} interact with the slow wave propagating along the device axis with the phase velocity close to v_{z0}



When the electron velocity slightly exceeds the wave phase velocity, the wave withdraws a part of the beam energy. This leads to amplification of the wave.

Backward-wave oscillators (BWOs)



In BWOs, there is the synchronism between electrons and the positive phase velocity of the wave, but the group velocity is negative that means that the EM energy propagates towards the cathode. (Internal feedback loop) Then, this wave is reflected from the cutoff cross-section and moves towards the output waveguide without interaction with the beam.

BWO driven GW-radar

Nanosecond Gigawatt Radar (NAGIRA) was built by Russians For the U.K.
Radar is driven by an X-band, relativistic (0.5 MV) BWO: 10 GHz, 0.5 GW, 5 ns pulse, 150 Hz rep frequency

Short pulse (5 nsec) – large instantaneous bandwidth – possibilities to detect objects with antireflection coating

GEC-Marconi and the UK Ministry of Defense

Pasotron: Plasma-Assisted BWO

Fig 1

Demonstrated at the U. Md. to operate with high efficiency (~50%) without external magnet or filament power supply.Could be developed as compact lightweight airborne source.

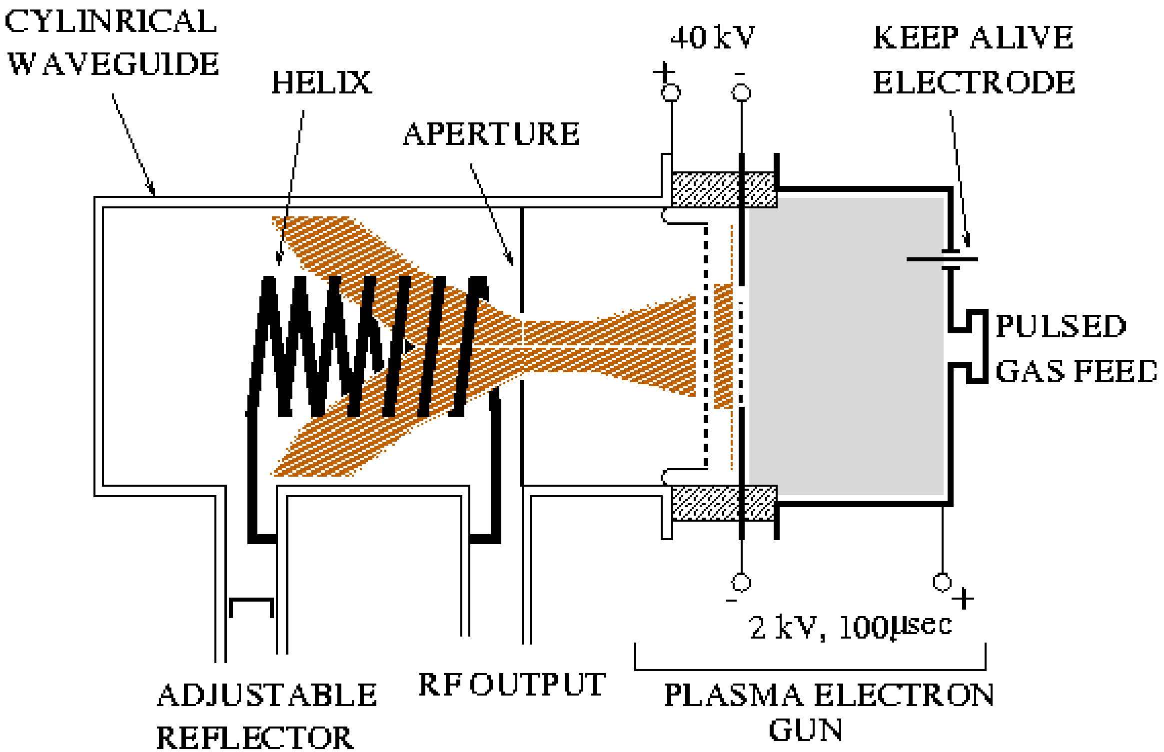
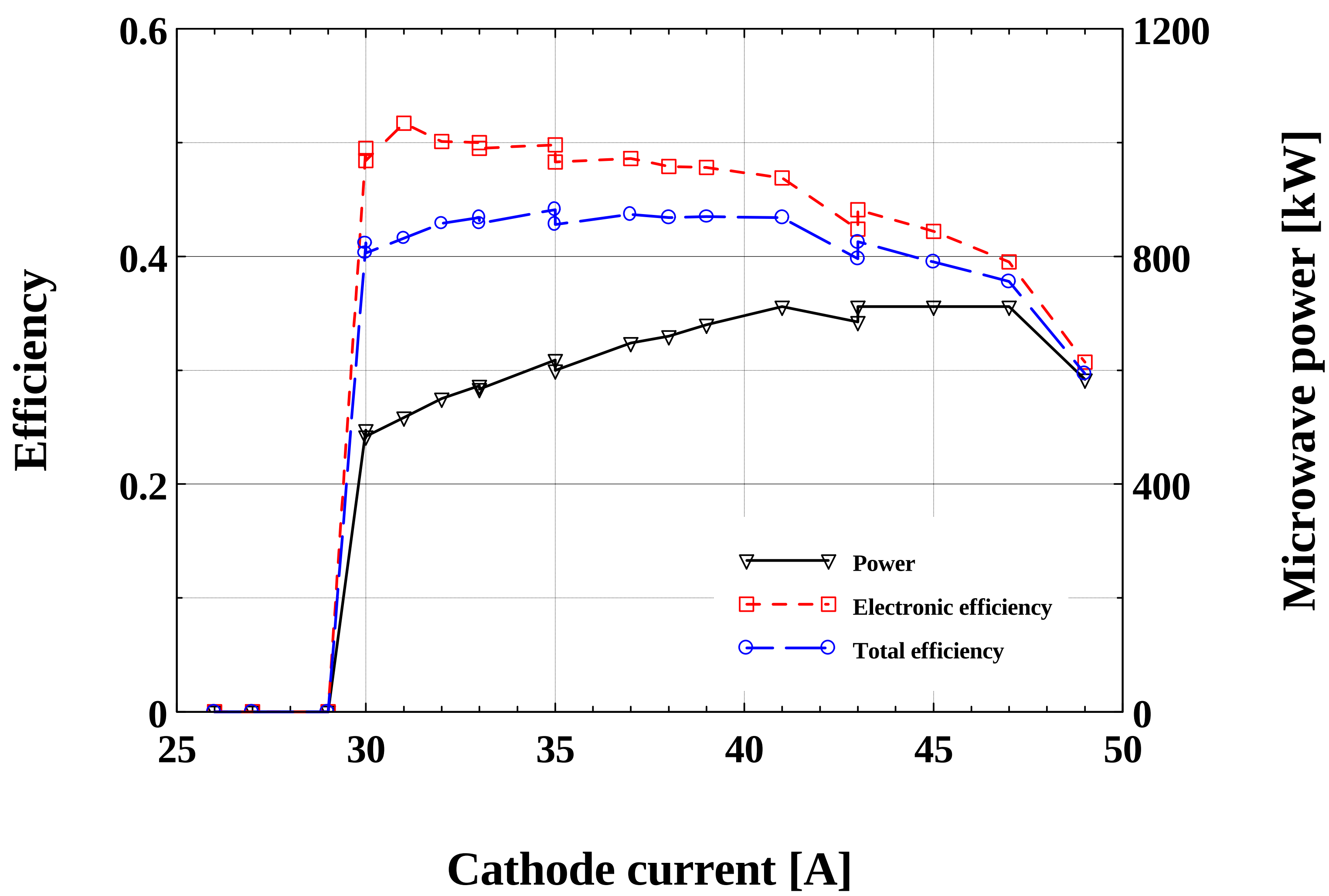
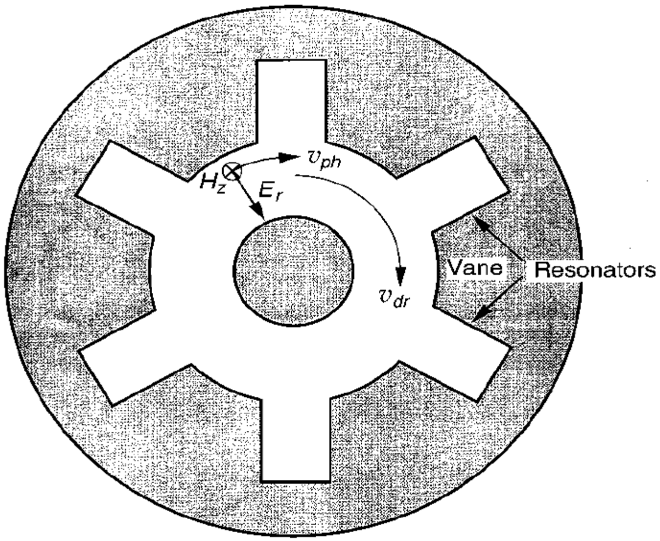


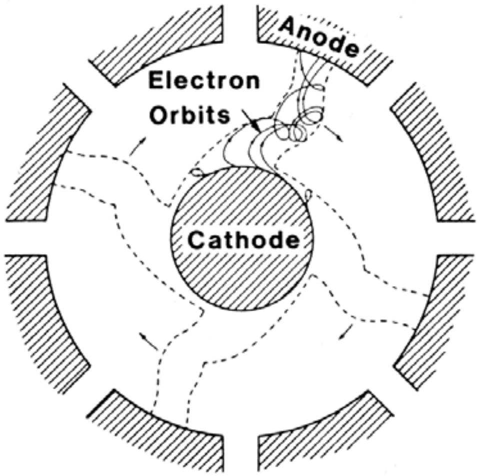
Fig. 5

Pasotron Power and Efficiency



Magnetrons





Electron spokes rotate synchronously with the EM wave rotating azimuthally

1) Drift velocity of electrons in crossed (E and H) fields is close to the phase velocity of the wave in the azimuthal direction – Cherenkov synchronism. ***Buneman-Hatree resonance condition***
2) Diameter of Larmor orbit should be smaller than the gap between cathode and anode. ***Hull cutoff***
3) All dimensions scale with the wavelength - $P(\lambda)$

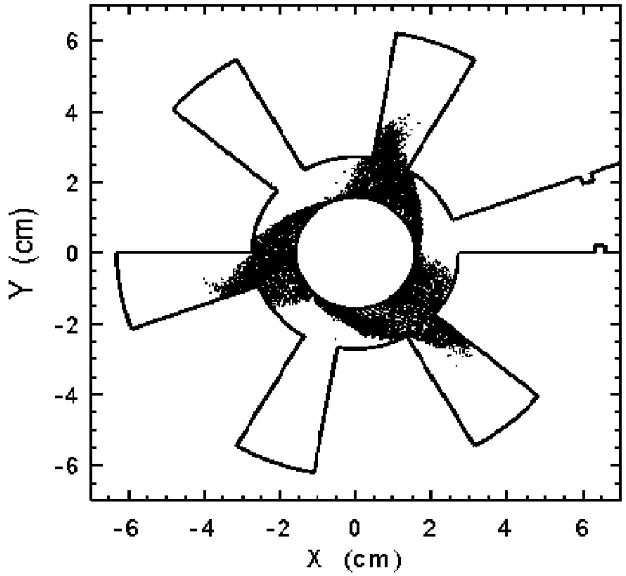
Relativistic Magnetrons

- **Many experiments with power from 10's o MW to GW have experienced pulse shortening.**

- **New simulations indicate a possible fix to operate at the several GW level without pulse shortening, at perhaps 50% source efficiency.**
  - **Nature of cathode may improve performance**

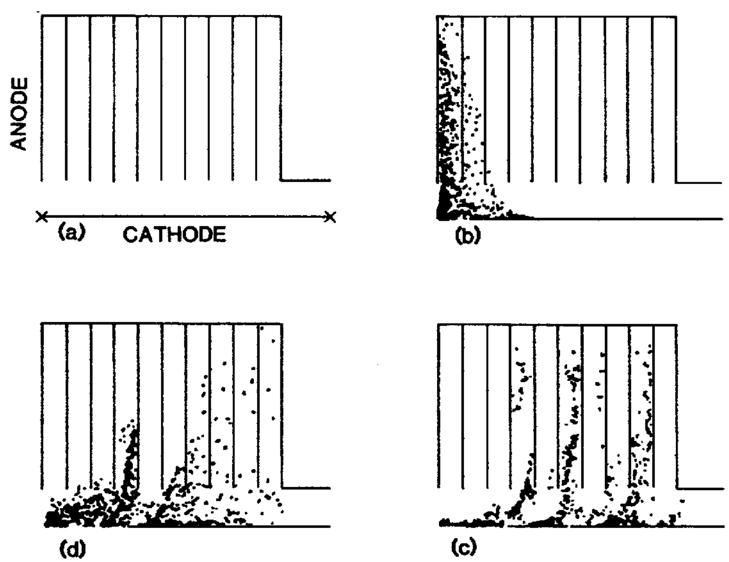
- **Simulations indicate a very small window in parameter space may exist for proper operation.**

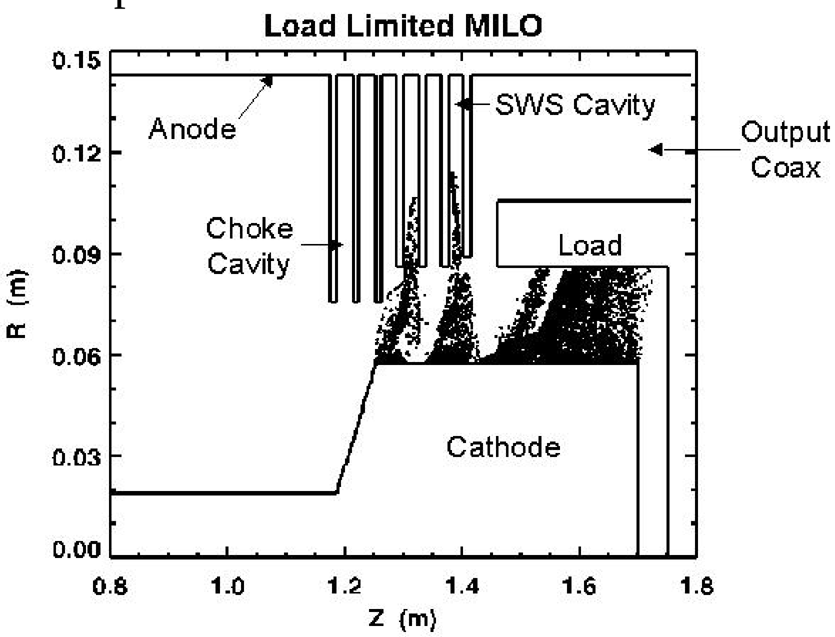
- **<5% variation in voltage, current and magnetic field.**
  - **Parameter space recently improved to facilitate experiment**



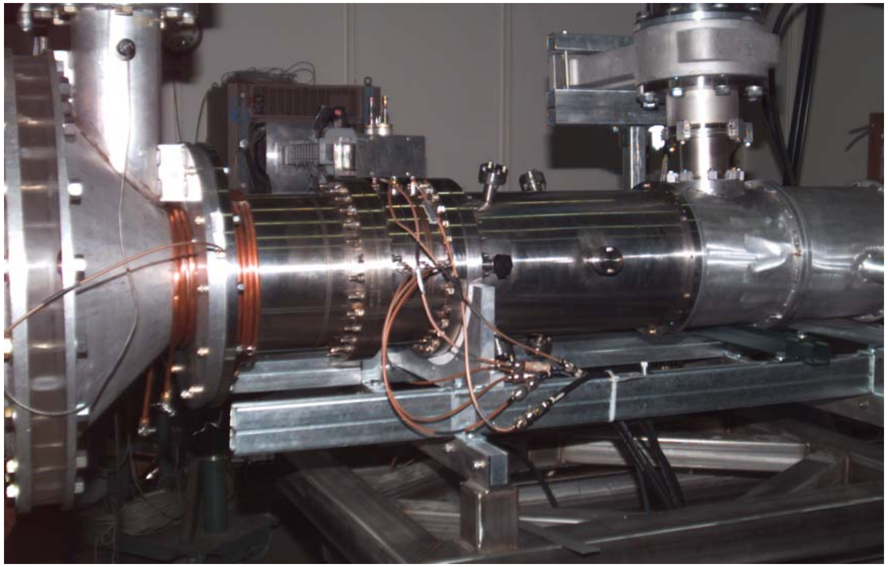
MILO Basics

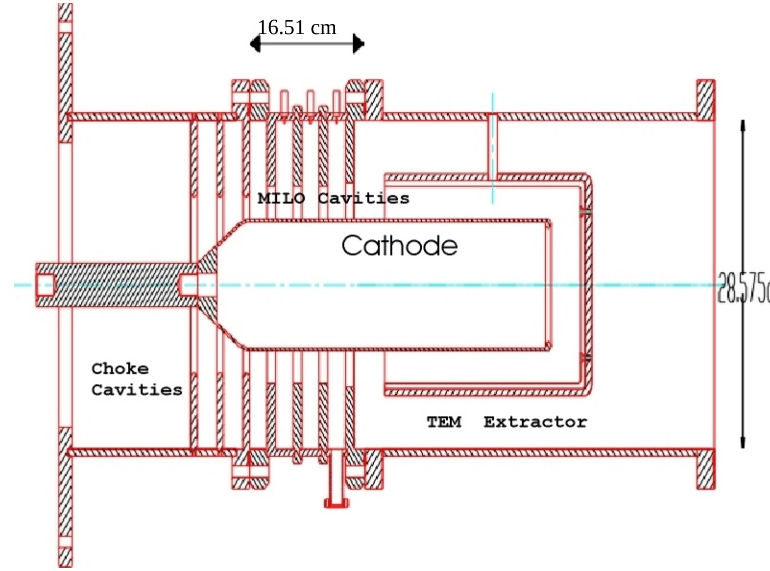
- Low impedance, high power
- Cross field source- applied E_r, self-generated B_θ, axial electron flow
- Device is very compact (No eternal focusing magnet)
- Efficiency is limited , due to power used to generate self-insulating magnetic field
- Cavity depth ~ $\lambda/4$
- Electron drift velocity $v_e >$ microwave phase velocity $v_\phi = 2\,p\,f$, where p is the axial periodicity of the structure and f is the microwave frequency
- Microwave circuit is eroded by repeated operation





The MILO/HTMILO (AFRL)





•Initial work developed the mode of extraction and introduced the choke section

•Power level up to 1.5 GW, mode competition and short Rf pulse

•Our first experiment to use brazed construction

•Observed loss of magnetic insulation when emission occurred under the choke vanes

•Shifted cathode 5 cm downstream, reducing field stress under choke vanes, tripled pulse length, 2 GW, 330 J

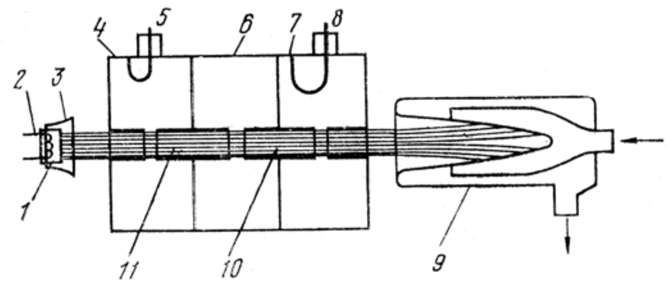
•Pulse power increased from 300 nsec to 600 nsec

•Present work on increasing power to 3 GW for up to 500 nsec, tuning last vane < 1 cm raises power from 1 to 3 GW

•Obtained a 400 nsec constant impedance at 450 kV

Sources of Coherent Transition Radiation

The best known source of coherent TR is the klystron (Varian brothers, 1939)

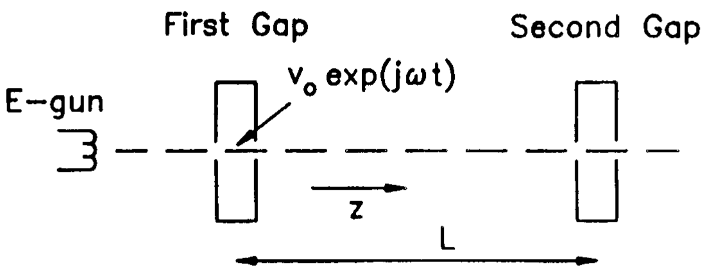


Schematic of a 3-cavity klystron:
1- cathode, 2 – heater, 3-focusing electrode, 4 – input cavity, 5 – input coaxial coupler, 6 – intermediate cavity, 7 – output cavity, 8 – output coaxial coupler, 9 – collector, 10, 11 – drift sections

Klystrons are also known as velocity modulated tubes: initial modulation of electron energies in the input cavity causes due to electron ballistic bunching in drift region following this cavity the formation of electron bunches, which can produce coherent radiation in subsequent cavities.

Klystron Basics

Classical Klystron Devices



•Electron dynamics are all single particle

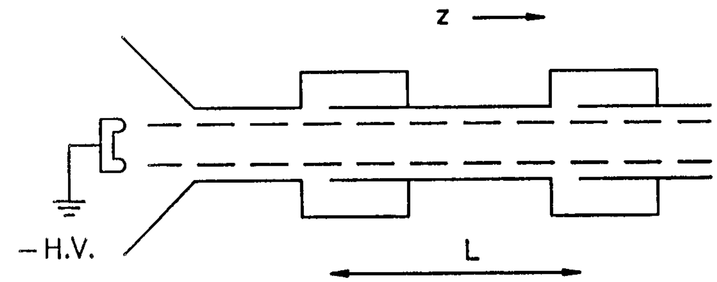
•No collective effects

•Microwave voltage induces a velocity modulation; electron beam drift allows for density modulation

•Gap voltages are limited by breakdown electric fields

•Electron beam transverse dimension < λ

Intense Beam Klystron



•Electron dynamics are not single particle

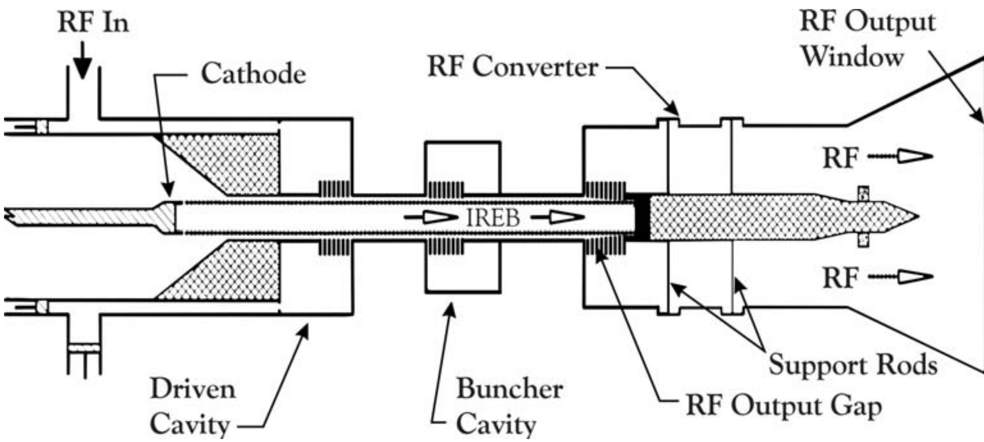
•Collective effects are critical

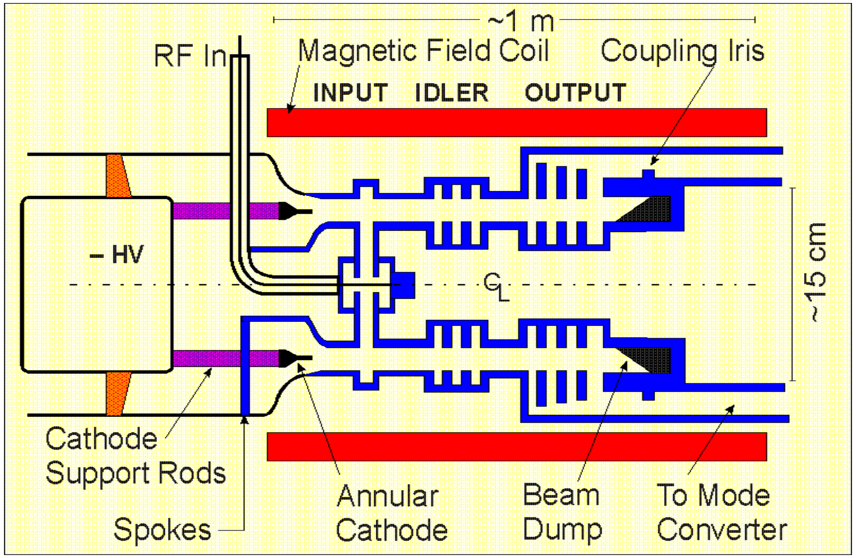
•Space charge potential energy is a large fraction of the total energy

•Beam current is large enough that gating occurs at the modulation gaps

Relativistic Klystron Amplifiers, RKAs, and Multiple Beam Klystrons, MBKs

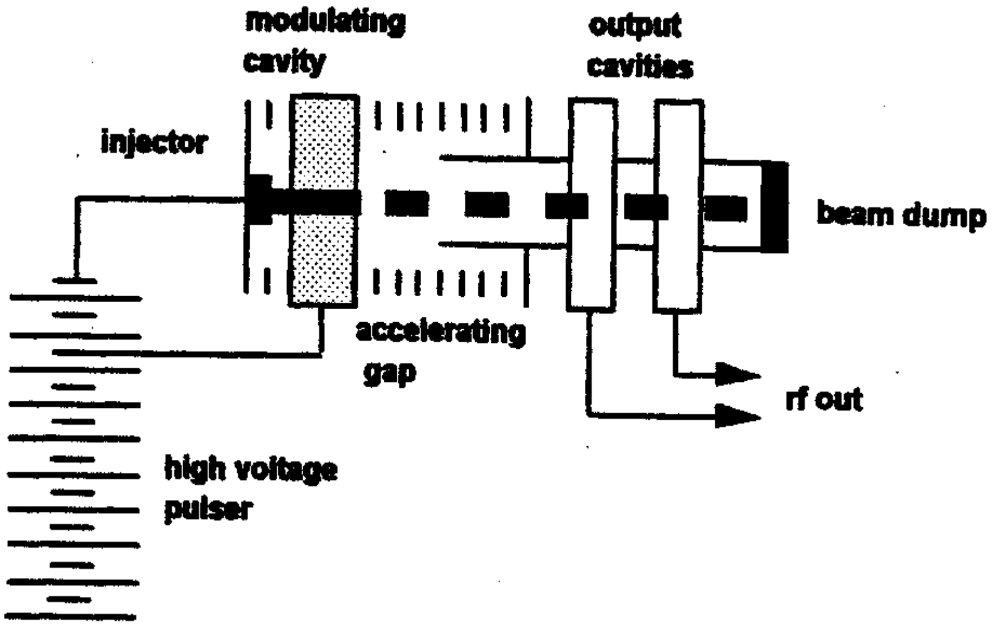
- NRL Experiment- 15 GW, 100 nsec, 1.3 GHz
- First experiment to use non-linear space charge effect for beam modulation
- NRL proposed Triaxial concept. This is an example of a Multiple Beam Klystron (MBK). Other MBKs use separate drift tubes but common cavities
- MRC proved the basic physics- 400 MW, 800 nsec, 11 GHz





SuperReltron

- Requires 1 MV(150/850 kV divider)/ ~2 kA pulse power
- pulse length 200 nsec to 1 μsec
- 600 MW, >200 J radiated
- demonstrated 50% efficiency
- extraction in TE_{10} rectangular mode
- 10's of pulses per second



Summary of HPM Sources
RF Characteristics

Device	Frequency	Peak Power	Pulse Length	Energy	Reprate	Efficiency
CPI/SLAC Klystron	2.998 GHz	150MW	3 μs	0.45 kJ	60 Hz	45%
Thomson CSF TH 1801 MBK	1.3 GHz	10 MW	1.5 ms	15 kJ	10 Hz†	70%
NRL RKA	1.3 GHz	6 GW	100 ns	0.6 kJ	S.S.	?
	1.3 GHz	3 GW	100 ns	0.3 kJ	S.S.	50%
MRC/AFRL RKA	1.3 GHz	1 GW	1 μs	1 kJ	1 Hz	45%
SLAC MBK Design	1.5 GHz	2 GW	1 μs	2 kJ	10 Hz	50%
Titan Reltron (Thermionic)	2.85 GHz	25 MW	2 μs	0.05 kJ	10 Hz	50%
Titan Super Reltron #1	0.7–1.0 GHz	0.4 GW	400 ns	0.16 kJ	10 Hz	50%
Titan Super Reltron #2	1.0–1.5 GHz	0.35 GW	400 ns	0.14 kJ	10 Hz	50%
AFRL MILO	1.2 GHz	1.5 GW	600 ns	0.35 kJ	S.S.	5%/11%
	1.2 GHz	300 MW	600 ns	0.18 kJ	S.S.	3%/7%
Maxwell (PI) Magnetron	1.15–1.3 GHz	0.5 GW	75 ns	0.038 kJ	100 Hz (10s)	13%
Maxwell (PI) Magnetron	2.4–3.3 GHz	0.5 GW	75 ns	0.038 kJ	100 Hz (10s)	13%
CPI VMS-1873 Magnetron	2.846 GHz	50 MW	600 ns	0.03 kJ	10 Hz	65%
	2.846 GHz	40 MW	1 μs	0.04 kJ	250 Hz (1s)	60%
	2.846 GHz	30 MW	1 μs	0.03 kJ	150 Hz	60%
	2.846 GHz	30 MW	5 μs	0.15 kJ	10 Hz	60%
CTL PM-600L Magnetron	915 MHz	0.6 MW	5 ms	3 kJ	100 Hz‡	>90%
	915 MHz	0.6 MW	330 μs	0.2 kJ	1500 Hz‡	>90%

†150 kW avg. ‡300 kW avg. Designs and devices under construction in red.

Non-Developmental RF Threat Demo

{Filmed/Live at NSWCDD, VA in 1998}

Demonstrate *TRUE* capabilities of non-developmental RF threats to equipment [Requested by OSD(C3I)]

ARL L-Band Source:
- 2 MW peak power
- 2 μs pulse width
- 300 Hz repetition
- 13 dB gain horn
- 1.3 GHz

NSWCDD Bow-Tie Antenna
- 150 kV Marx generator

NSWCDD Discone Antenna
- 300 kV Marx generator

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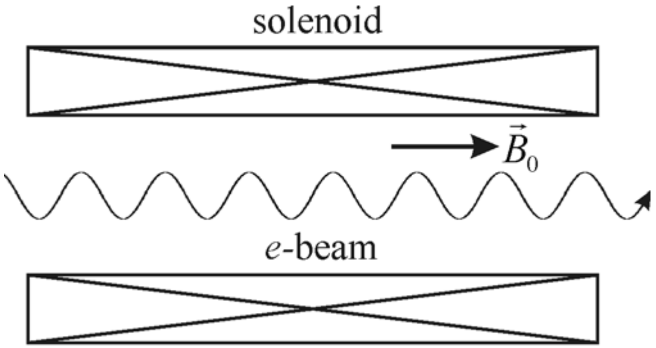
Sources of coherent radiation from the beams of oscillating electrons

I. Cyclotron resonance masers, CRMs (gyrotrons)

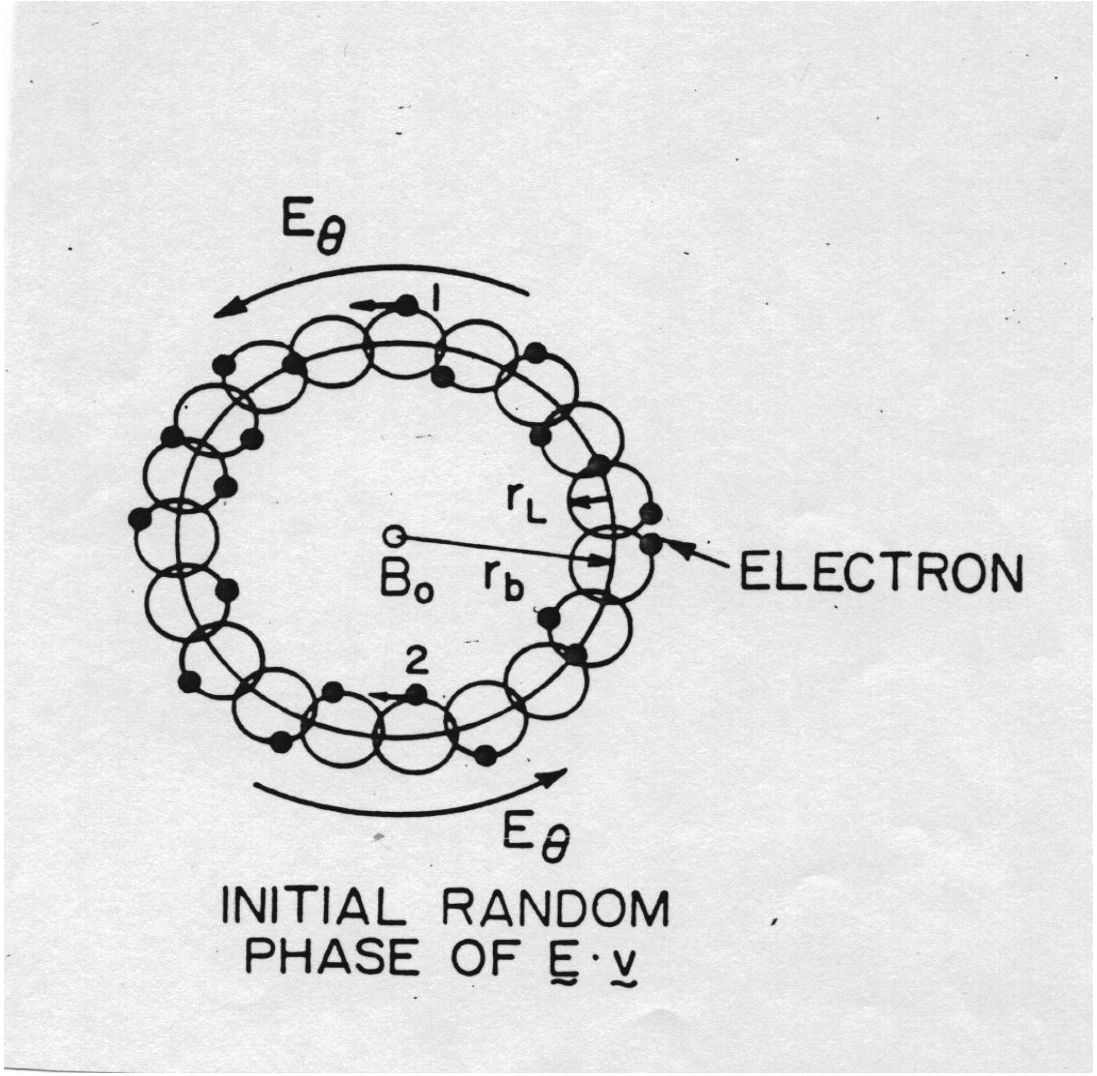
Electrons oscillate in a constant magnetic field



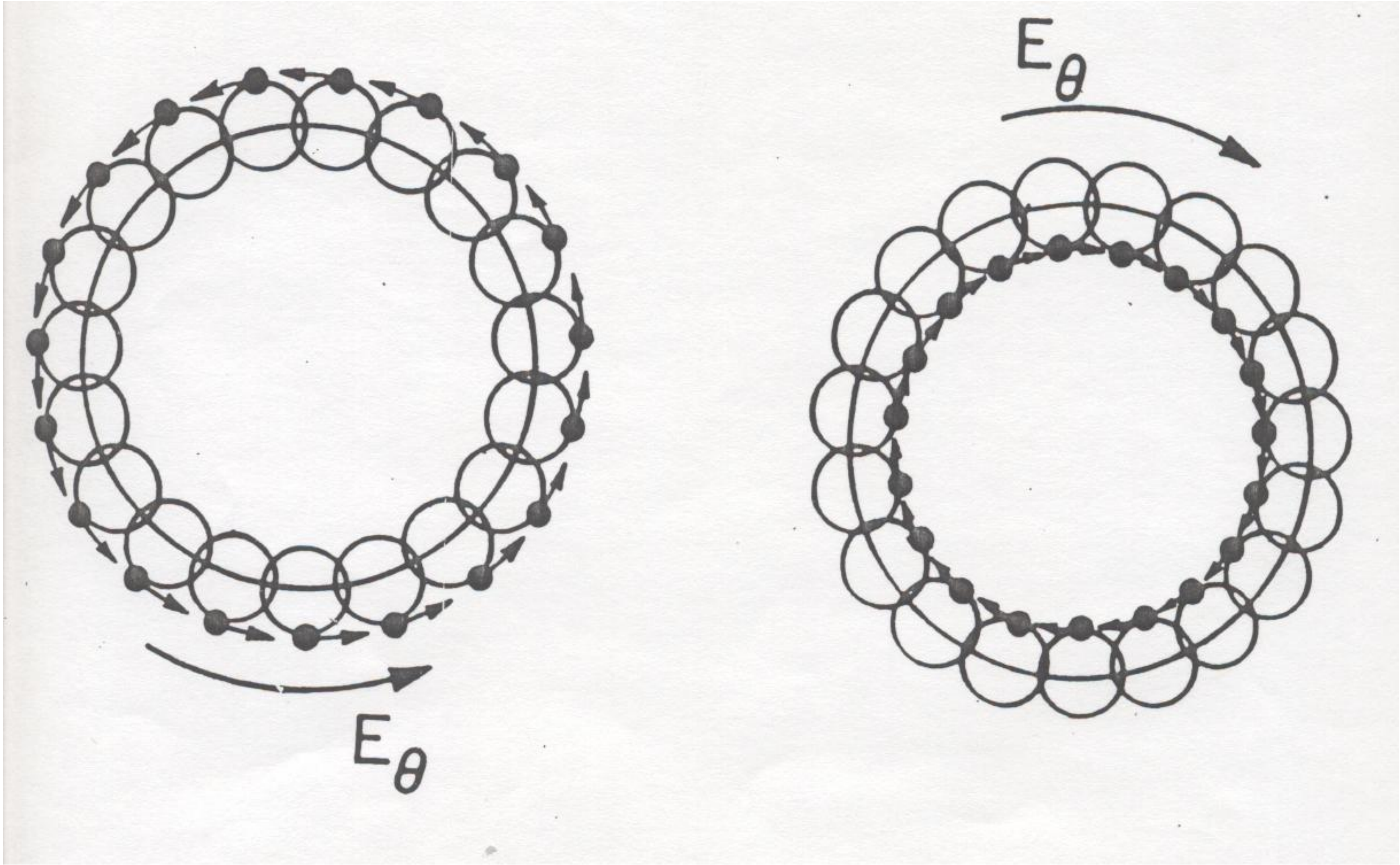
In the cyclotron resonance condition $\omega - k_z v_z = s\Omega$

$\Omega = eB_0 / mc\gamma$ **Electron bunching is caused by the <u>relativistic effect</u> – relativistic dependence of electron mass on energy**

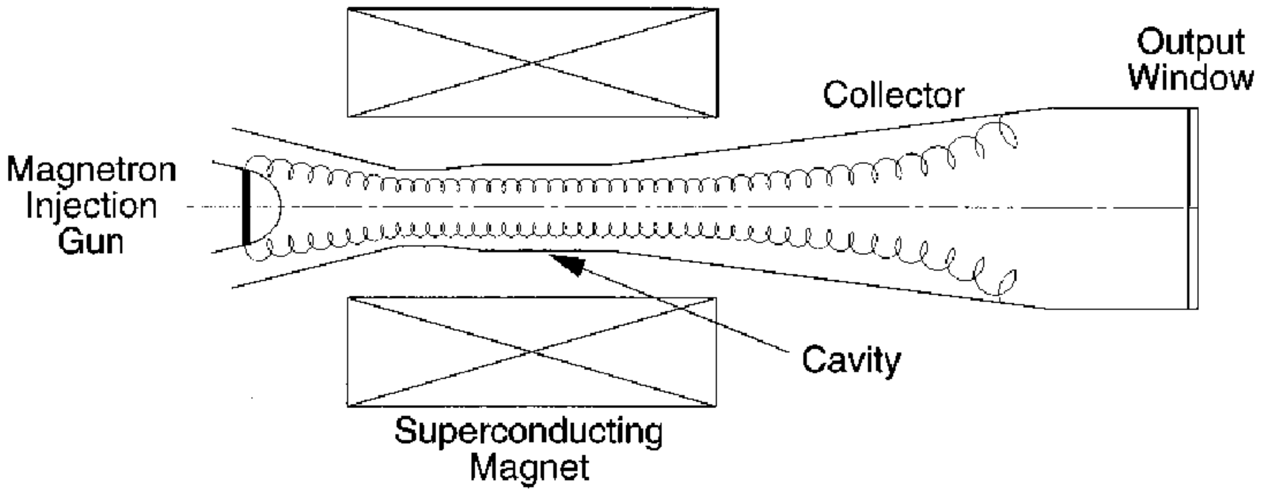
Cross-section of Gyrotron e-beam (before phase bunching) $\omega_c = eB/\gamma m$



Cross-section of Gyrotron e-beam (after phase bunching)



Gyrotrons



Gyrotron is a specific configuration of CRM comprising a magnetron-type electron gun and an open microwave structure for producing high-power millimeter-wave radiation

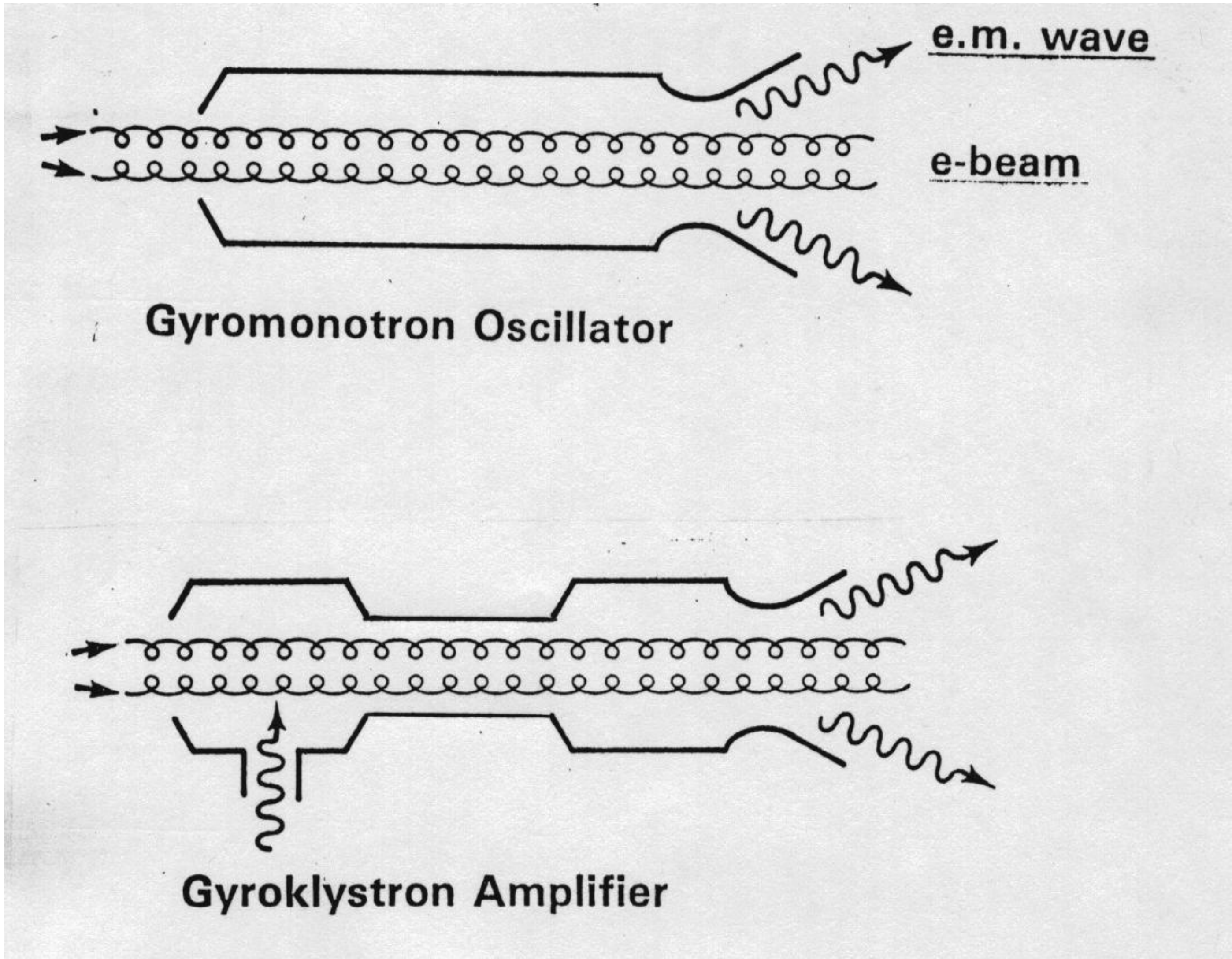
Gyrotron Principles

* Hollow beam of spiralling electrons used
* Resonance at the electron cyclotron frequency (or 2nd harmonic) matched to frequency of high order cavity mode

  (discrimination against spurious modes)
* Electrons bunch in phase of their cyclotron orbits
* Transverse dimensions of cavities and

  e-beam may be much larger than the wavelength and high power operation may be extended to very high frequency

Schematic of Gyrotron Oscillator and Amplifier Circuits



Gyromonotron

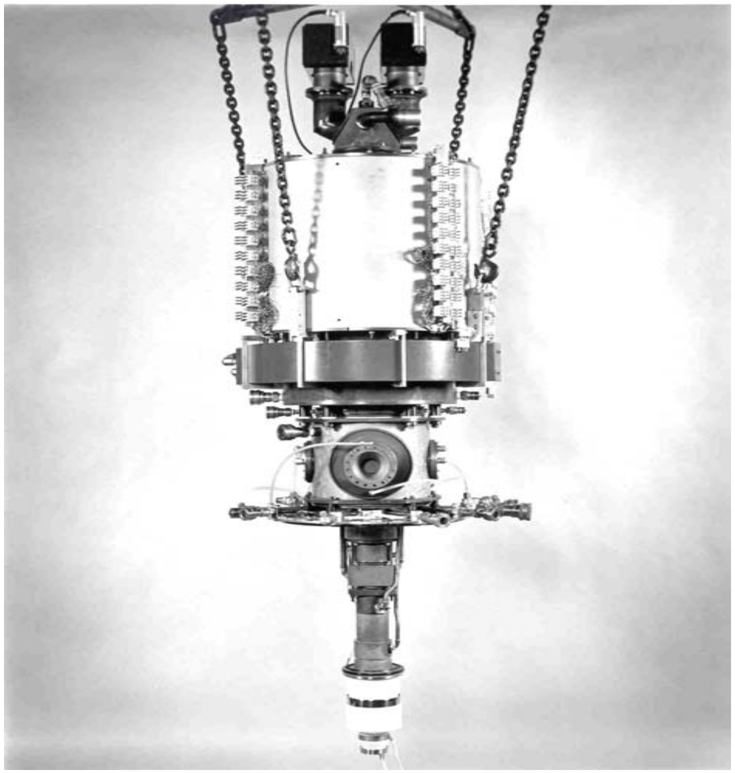
Due to
(a) the cyclotron resonance condition, which provides efficient interaction of gyrating electrons only with the modes of a microwave structure whose Doppler-shifter frequencies are in resonance with gyrating electrons, and
(b) high selectivity of open microwave circuits,

gyrotrons can operate in very high-order modes (e.g. $TE_{22,6}$)

Interaction volume can be much larger than λ^3

Gyrotrons can handle very high levels of average power – MWs CW at ~2 mm wavelength

Gyromonotron (cont.)



110 GHz, 1 MW, CPI Gyrotron (about 2.5 m long)

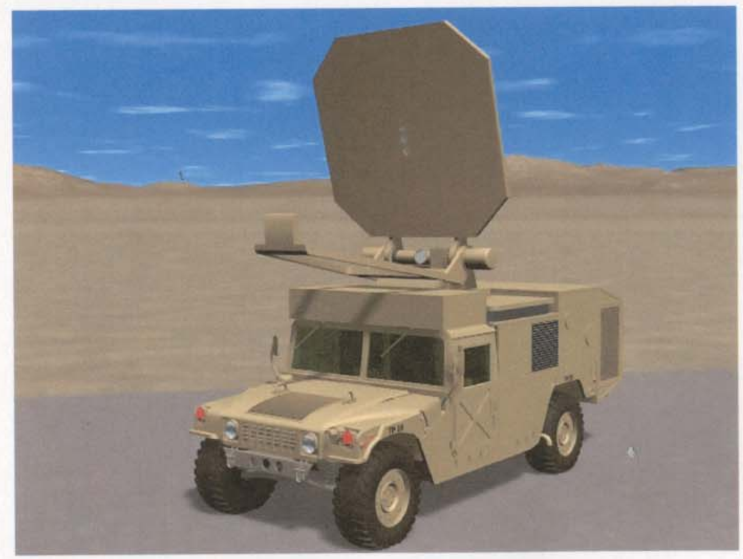
Active denial technology hardware demonstration (fromUS AF web page)

Technology/System Description

Name: Active Denial Technology

Description:

- ***Nonlethal antipersonnel directed energy weapon***
- ***Long range, lightspeed, line-of-sight, deep magazine***
- ***Energy beam creates a directional sensation of pain, causing repel without damage***



Potential Applications:

- ***Area Delay/Denial***
- ***Force Protection***

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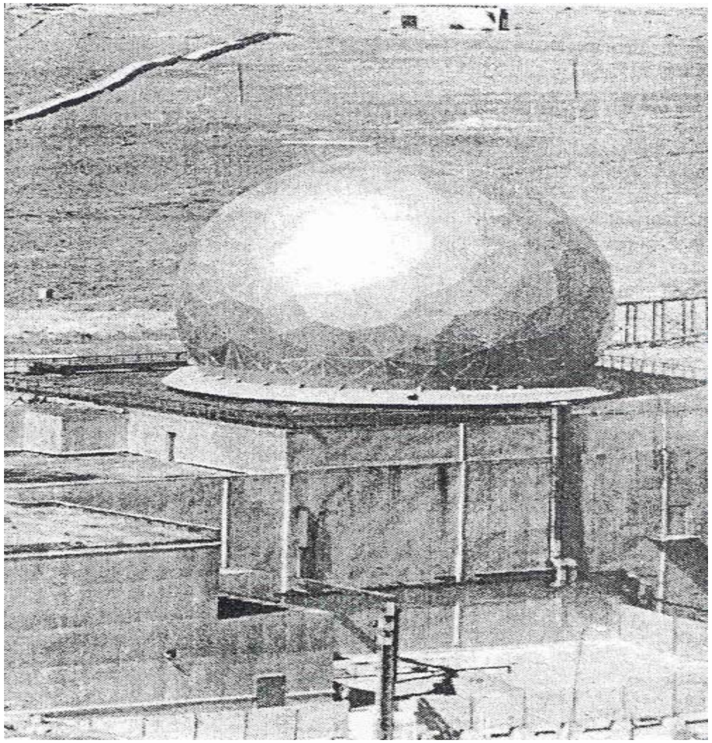
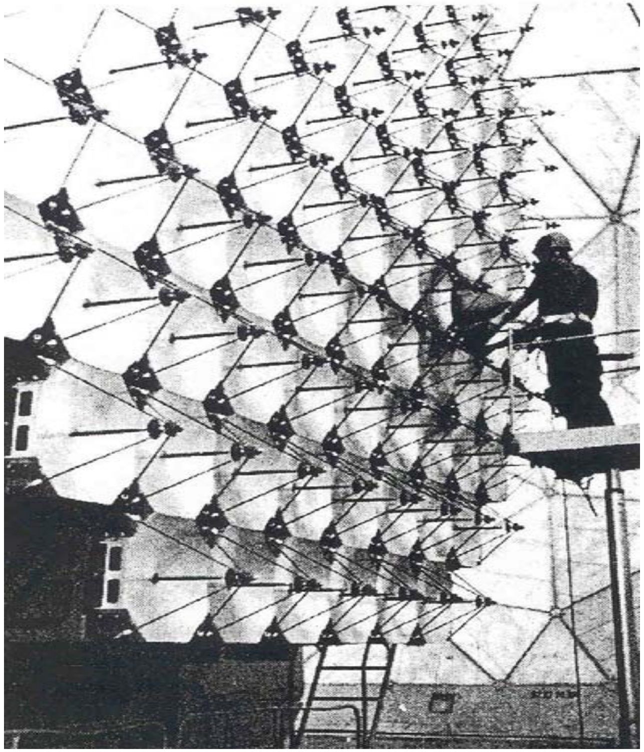
Gyroklystrons

In contrast to gyrotron oscillators discussed above, gyroklystrons are amplifiers of input signals. Amplifiers produce phase-controlled radiation, Thus, they can be used in communication systems, radars and other applications requiring the phase control (viz., phase arrays).

NRL (in collaboration with CPI and UMd):
W-band (94 GHz) gyroklystrons and gyrotwystrons,
100 kW peak, 10 kW average power

Gyroklystrons (cont.)





Russian 1 MW, Ka-band (34 GHz) radar "Ruza",

Two 0.7 MW gyroklystrons (Tolkachev et al., IEEE AES Systems Mag., 2000)

Gyroklystrons (cont.)

NRL W-band gyroklystron and “WARLOC” radar

WAVEGUIDES

Assume uniform cross-section and wave propagation along z-axis as $e^{-\alpha z}\cos(\omega t - \beta z)$

(a) Transmission Lines or 2-Conductor Waveguides (e.g. Co-axial Cable)

(b) Hollow Pipe Waveguide (Rectangular and Circular)

Dispersion Curves

In 2-conductor transmission lines such as co-axial cable, a TEM wave may propagate with no axial field components and dispersion relation

$$\omega^2 = \beta^2 /(\mu\varepsilon) = \beta^2 c^2 /\varepsilon_r$$

where ε_r is the relative permittivity of the dielectric between the 2 coductors and $c = 3 \times 10^8$ m/s is the speed of light in vacuum.

In hollow pipe waveguide no TEM wave can propagate. The modes of propagation are either TE (axial magnetic field present) or TM (axial electric field present) and the dispersion relation for each mode is

$$\omega^2 = \beta^2 c^2 + \omega_c^2$$

where $f_c = \omega_c /2\pi$ is called the cutoff frequency (usually a different value for each mode) and we have assumed that the waveguide is filled with gas for which to good approximation $\varepsilon_r = 1$. Note that for $\omega < \omega_c$ no real values of β are possible and the wave will not propagate.

Co-axial Cable

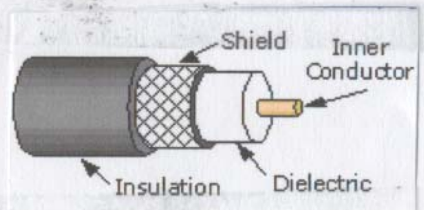
Wave propagates in dielectric beween inner and outer conductors
a = inner radius; b = outer radius of dielectric.
μ,ε,σ pertain to the dielectric; μ_c,σ_c pertain to the conductors
$R_s = (\pi f \mu_c/\sigma_c)^{1/2}$; Characteristic Impedance $R_o = (L'/C')^{1/2}$

Radial E-Fields

Cylindrical H-Fields

No cutoff for lowest (TEM) wave

E_z, H_z = 0

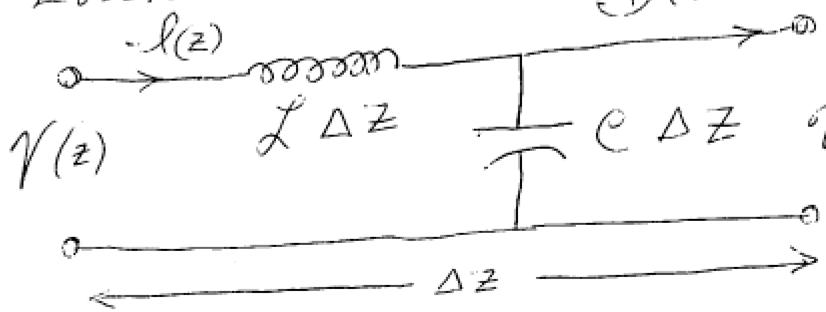


Parameter	Coaxial	
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	Resistance /length Ohms/meter
L'	$\frac{\mu}{2\pi}\ln(b/a)$	Inductance/length Henries/meter
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	Conductance/length Siemens/meter
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	Capacitance/length Farads/meter

Power Transmitted
$P = 0.5\ V^2 / R_o$
Where V is peak voltage between the conductors

Equivalent Ckts. of Incremental Lengths of Generalized Transmission Lines

(a) Lossless Line



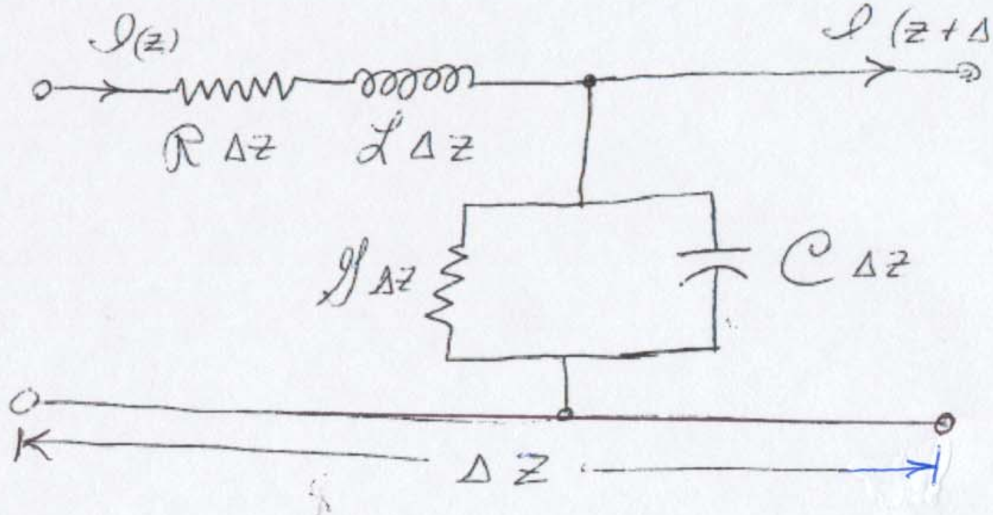
$$V(z+\Delta z) - V(z) = -j\omega \mathcal{L} \Delta z \, I(z)$$

$$\frac{\Delta V}{\Delta z} = -j\omega \mathcal{L} I(z) \xrightarrow[\Delta z \to 0]{} \boxed{\frac{dV}{dz} \quad -j\omega \mathcal{L} I(z)}$$

$$I(z+\Delta z) - I(z) = -j\omega C \Delta z \, V(z+\Delta z)$$

$$\frac{\Delta I}{\Delta z} = -j\omega C \, V(z+\Delta z) \xrightarrow[\Delta z \to 0]{} \boxed{\frac{dI}{dz} = -j\omega C \, V(z)}$$

Equiv. CKt of Lossy Transmission Line



$$\mathcal{L} = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\mathcal{C} = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$$

$$\frac{dV}{dz} = -(\mathcal{R} + j\omega\mathcal{L})\, I$$

$$\frac{dI}{dz} = -(\mathcal{G} + j\omega\mathcal{C})\, V$$

Helmholtz Eq. for Lossy Transmission Line

$$\frac{d^2 V}{dz^2} = \gamma^2 V \quad \text{or} \quad \frac{d^2 I}{dz^2} = \gamma^2 I$$

Same form as for lossless line

but now γ is complex $\Rightarrow$ wave attenuation

For lossless line, $\gamma = j\beta = j\omega\sqrt{\mathcal{L}\mathcal{C}}$

For lossy line, $\gamma = \alpha + j\beta = \sqrt{(R + j\omega\mathcal{L})(\mathcal{G} + j\omega\mathcal{C})}$

Characteristic Impedance:

For lossless line, $Z_0 = \sqrt{\mathcal{L}/\mathcal{C}}$

For lossy line, $Z_0 = R_0 + jX_0 = \sqrt{\dfrac{R + j\omega\mathcal{L}}{\mathcal{G} + j\omega\mathcal{C}}}$

$$P_{AV} = \frac{1}{2}\,\mathrm{Re}\,(V \times I^*) = \frac{1}{2}\,\mathrm{Re}\,(I Z_0 \times I^*) = \frac{1}{2}|I|^2 R_0$$

Low Loss Approximation

$$\mathcal{R} \ll \omega\mathcal{L} \quad \text{and} \quad \mathcal{G} \ll \omega\mathcal{C}, \quad Z_0 \approx \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = R_0$$

(esp. likely to be satisfied for large ω)

$$\gamma = \alpha + j\beta = \sqrt{(\mathcal{R}+j\omega\mathcal{L})(\mathcal{G}+j\omega\mathcal{C})}$$

$$= \sqrt{j\omega\mathcal{L}\left(1+\frac{\mathcal{R}}{j\omega\mathcal{L}}\right) j\omega\mathcal{C}\left(1+\frac{\mathcal{G}}{j\omega\mathcal{C}}\right)}$$

$$= j\omega\sqrt{\mathcal{L}\mathcal{C}}\left(1+\frac{\mathcal{R}}{j\omega\mathcal{L}}\right)^{1/2}\left(1+\frac{\mathcal{G}}{j\omega\mathcal{C}}\right)^{1/2}$$

$$\cong j\omega\sqrt{\mathcal{L}\mathcal{C}}\left(1+\frac{\mathcal{R}}{2j\omega\mathcal{L}}\right)\left(1+\frac{\mathcal{G}}{2j\omega\mathcal{C}}\right)$$

$$= j\omega\sqrt{\mathcal{L}\mathcal{C}}\left(1 - j\left(\frac{\mathcal{R}}{2\omega\mathcal{L}} + \frac{\mathcal{G}}{2\omega\mathcal{C}}\right)\right)$$

$$\therefore \beta \cong \omega\sqrt{\mathcal{L}\mathcal{C}} = \omega\sqrt{\mu\epsilon} = \omega(\epsilon_r/c)$$

$$\alpha = \frac{1}{2}\left[\mathcal{R}\sqrt{\frac{\mathcal{C}}{\mathcal{L}}} + \mathcal{G}\sqrt{\frac{\mathcal{L}}{\mathcal{C}}}\right] = \frac{1}{2}\left[\frac{\mathcal{R}}{R_0} + \mathcal{G}R_0\right]$$

Properties of Coax Cable Designed for High Voltage Pulse Operation

Cable designation: RG-193/U
o.d.: 2.1 inches
Characteristic Impedance: R_o = 12.5 Ohms
Maximum Operating Voltage: V_{max} = 30,000 Volts

Maximum Power Transmitted: $P_P = 0.5\, V_{max}^2 / R_o$ = 18 Megawatts

This value of peak power is inadequate for many HPM sources. Rectangular or circular waveguides designed for f = 1 GHz are larger in transverse dimensions than co-axial cable and can be filled with pressurized, high dielectric strength gas so that they can transmit higher peak power

Hollow Pipe Waveguides are Analyzed Using Maxwell,s Equations

Electric field $\vec{E}$ and magnetic field $\vec{H}$ can be found from Maxwell's equations: *(in phasor form)*

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \vec{J}$$

$$\nabla \bullet \varepsilon\vec{E} = \rho \qquad \nabla \bullet \mu\vec{H} = 0$$

Permeability μ and permittivity ε are properties of the medium

Current density $\vec{J}$ and charge density ρ are sources

Assume sinusoidal steady-state with frequency ω; use phasor notation so that $\vec{E}(t) = \mathrm{Re}(\vec{E}e^{j\omega t})$ etc.

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x \frac{\partial V}{\partial x} + \mathbf{a}_y \frac{\partial V}{\partial y} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \mathbf{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \mathbf{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r \frac{\partial V}{\partial r} + \mathbf{a}_\phi \frac{\partial V}{r \partial \phi} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r \partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R \frac{\partial V}{\partial R} + \mathbf{a}_\theta \frac{\partial V}{R \partial \theta} + \mathbf{a}_\phi \frac{1}{R \sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R \frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]$$
$$+ \mathbf{a}_\theta \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$
$$+ \mathbf{a}_\phi \frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Some Useful Vector Identities

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$\nabla(\psi V) = \psi \nabla V + V \nabla \psi$$

$$\nabla \cdot (\psi \mathbf{A}) = \psi \nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla \psi$$

$$\nabla \times (\psi \mathbf{A}) = \psi \nabla \times \mathbf{A} + \nabla \psi \times \mathbf{A}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \cdot \nabla V = \nabla^2 V$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

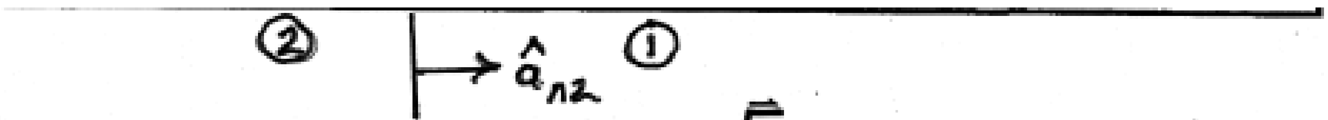
$$\nabla \times \nabla V = 0$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\int_V \nabla \cdot \mathbf{A}\, dv = \oint_S \mathbf{A} \cdot d\mathbf{s} \qquad \text{(Divergence theorem)}$$

$$\int_S \nabla \times \mathbf{A} \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \qquad \text{(Stokes's theorem)}$$

Boundary Conditions



At boundary, tangential component of $\vec{E}$ is continuous;
i.e. $E_{t1} = E_{t2}$

Normal component of $\vec{D} = \varepsilon\vec{E}$ is discontinuous by amt. of surface charge; i.e., $D_{n2} - D_{n1} = \rho_s$

$$\hat{a}_{n2} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

Tangential component of $\vec{H}$ is discontinuous by amt. of surface current; i.e. $H_{t1} - H_{t2} = J_s$ or at the surface of a perfect conductor, $\vec{a}_n \times \vec{H} = -\vec{J}_s$

$$\hat{a}_{n2} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

Normal component of $\vec{B} = \mu\vec{H}$ is continuous; i.e. $B_{n1} = B_{n2}$

Source Free Regions

Inside waveguide there are no sources (i.e., $\boldsymbol{J} = 0$ and $\rho = 0$).
Then, Maxwell.s curl equations become symmetric

$$\nabla \times \boldsymbol{E} = j\omega\mu\boldsymbol{H} \quad \text{-----} \ (1), \qquad \nabla \times \boldsymbol{H} = -j\omega\varepsilon\boldsymbol{E} \quad \text{------} \ (2)$$

Taking the curl of Eq. (1) and substituting from Eq. (2) gives

$$\nabla \times \nabla \times \boldsymbol{E} = \omega^2 \mu\varepsilon \boldsymbol{E}$$

Next use the vector identity $\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$
and the fact that in a sourceless uniform region $\nabla \cdot \boldsymbol{E} = \rho/\varepsilon = 0$
to get

$$\nabla^2 \boldsymbol{E} + \omega^2 \mu\varepsilon \boldsymbol{E} = 0$$

This is the homogeneous Helmholtz equation in ***E*.**
A similar equation could have been derived for ***H*.**

Helmholtz Equation

The fields will obey a homogeneous Helmholtz Eq.

i.e., $\nabla^2 \vec{\mathcal{E}} + k^2 \vec{\mathcal{E}} = 0$ -- (1a)

or $\nabla^2 \vec{\mathcal{H}} + k^2 \vec{\mathcal{H}} = 0$ -- (1b)

where $k^2 = \omega^2 \mu \epsilon$ and $\nabla^2 = \nabla_T^2 + \frac{\partial^2}{\partial z^2}$

∇_T^2 is the transverse part of the ∇^2 operator

e.g. in Cartesian co-ords., $\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

in circ. cylin. co-ords., $\nabla_T^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$

Then (1a) $\Rightarrow$ $\nabla_T^2 \vec{\mathcal{E}} + (\gamma^2 + k^2)\vec{\mathcal{E}} = 0$ -- (2a)

and (1b) $\rightarrow$ $\nabla_T^2 \vec{\mathcal{H}} + (\gamma^2 + k^2)\vec{\mathcal{H}} = 0$ -- (2b)

Scheme for Solving Helmholtz Eq.

Eqs. (2a) and (2b) are each equivalent to 3 scalar eqs.

We need to solve only 1 of these scalar p.d.e's

$$\nabla_T^2 \mathcal{E}_z + h^2 \mathcal{E}_z = 0 \quad \text{or} \quad \nabla_T^2 \mathcal{H}_z + h^2 \mathcal{H}_z = 0$$

(where $h^2 = \gamma^2 + k^2$)

depending on whether wave is TM or TE.

Then, other field components can be found from Maxwell's curl equs.

$$\nabla \times \vec{\mathcal{E}} = -j\omega\mu \vec{\mathcal{H}} \quad \text{and} \quad \nabla \times \vec{\mathcal{H}} = j\omega\epsilon \vec{\mathcal{E}}$$

these 2 vector equations are equivalent to 6 scalar equations

Expressions for finding transverse field components

Maxwell's curl equations in Cartesian co-ordinates are:

$$\frac{\partial \mathcal{E}_z}{\partial y} + \gamma \mathcal{E}_y = -j\omega\mu \mathcal{H}_x \quad \text{(3a)} \qquad \frac{\partial \mathcal{H}_z}{\partial y} + \gamma \mathcal{H}_y = j\omega\epsilon \mathcal{E}_x \quad \text{(4a)}$$

$$-\gamma \mathcal{E}_x - \frac{\partial \mathcal{E}_z}{\partial x} = -j\omega\mu \mathcal{H}_y \quad \text{(3b)} \qquad -\gamma \mathcal{H}_x - \frac{\partial \mathcal{H}_z}{\partial x} = j\omega\epsilon \mathcal{E}_y \quad \text{(4b)}$$

$$\frac{\partial \mathcal{E}_y}{\partial x} - \frac{\partial \mathcal{E}_x}{\partial y} = -j\omega\mu \mathcal{H}_z \quad \text{(3c)} \qquad \frac{\partial \mathcal{H}_y}{\partial x} - \frac{\partial \mathcal{H}_x}{\partial y} = j\omega\epsilon \mathcal{E}_z \quad \text{(4c)}$$

where we have assumed $\frac{\partial}{\partial z} \rightarrow -\gamma$

The 6 scalar equations above may be rearranged to express the four transverse field components in terms of the two axial (z) field components ($\mathcal{E}_z$, $\mathcal{H}_z$)

Expressions for transverse components (cont.)

$$\mathcal{H}_x = -\frac{1}{h^2}\left(\gamma \frac{\partial \mathcal{H}_z}{\partial x} - j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial y}\right) \quad \cdots (5a)$$

$$\mathcal{H}_y = -\frac{1}{h^2}\left(\gamma \frac{\partial \mathcal{H}_z}{\partial y} + j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial x}\right) \quad \cdots (5b)$$

$$\mathcal{E}_x = -\frac{1}{h^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial x} + j\omega\mu \frac{\partial \mathcal{H}_z}{\partial y}\right) \quad \cdots (5c)$$

$$\mathcal{E}_y = -\frac{1}{h^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial y} - j\omega\mu \frac{\partial \mathcal{H}_z}{\partial x}\right) \quad \cdots (5d)$$

For TM waves $\mathcal{H}_z = 0$ and equations simplify;

also, for TE waves $\mathcal{E}_z = 0$ and equations simplify.

For TM waves $\mathcal{H}_z = 0$, $\mathcal{E}_z \neq 0$

We need to solve $\nabla_T^2 \mathcal{E}_z + h^2 \mathcal{E}_z = 0$
subject to the boundary conditions of a particular waveguide

Then eq. (5a) $\Rightarrow$ $\mathcal{H}_x = \frac{j\omega\epsilon}{h^2} \frac{\partial \mathcal{E}_z}{\partial y}$ ... (6a)

(5b) $\rightarrow$ $\mathcal{H}_y = -\frac{j\omega\epsilon}{h^2} \frac{\partial \mathcal{E}_z}{\partial x}$ ... (6b)

(5c) $\rightarrow$ $\mathcal{E}_x = -\frac{\gamma}{h^2} \frac{\partial \mathcal{E}_z}{\partial x}$

(5d) $\rightarrow$ $\mathcal{E}_y = -\frac{\gamma}{h^2} \frac{\partial \mathcal{E}_z}{\partial y}$

$Z_{TM} = \frac{\mathcal{E}_x}{\mathcal{H}_y} = -\frac{\mathcal{E}_y}{\mathcal{H}_x} = \frac{\gamma}{j\omega\epsilon}$ Waveguide impedance

$\vec{\mathcal{H}} = \frac{1}{Z_{TM}} \left(\hat{a}_z \times \vec{\mathcal{E}}_t \right)$ Relation between transverse components

30–17

<u>For TE waves</u> $\mathcal{H}_z \neq 0, \ \mathcal{E}_z = 0$

Solve $\nabla^2 \mathcal{H}_z + h^2 \mathcal{H}_z = 0$

subject to boundary conditions of particular waveguide

Then, eq. (5a) $\Rightarrow \mathcal{H}_x = -\dfrac{\gamma}{h^2}\dfrac{\partial \mathcal{H}_z}{\partial x}$ ... (7a)

(5b) $\Rightarrow \mathcal{H}_y = -\dfrac{\gamma}{h^2}\dfrac{\partial \mathcal{H}_z}{\partial y}$ ... (7b)

(5c) $\Rightarrow \mathcal{E}_x = -\dfrac{j\omega\mu}{h^2}\dfrac{\partial \mathcal{H}_z}{\partial y}$ ... (7c)

(5d) $\Rightarrow \mathcal{E}_y = \dfrac{j\omega\mu}{h^2}\dfrac{\partial \mathcal{H}_z}{\partial x}$ ... (7d)

$$Z_{TE} = \frac{\mathcal{E}_x}{\mathcal{H}_y} = -\frac{\mathcal{E}_y}{\mathcal{H}_x} = \frac{j\omega\mu}{\gamma}$$ Waveguide impedance

$$\vec{\mathcal{E}} = -Z_{TE}\left(\hat{a}_z \times \vec{\mathcal{H}}\right)$$ Relation between transverse components

Eigenvalues

We will find that when we solve Helmholtz Eq. for $\mathcal{E}$ or $\mathcal{H}$ and apply boundary conditions that only certain discrete values of h are allowed

i.e. Solve: $\nabla_T^2 \mathcal{E}_z + h^2 \mathcal{E}_z = 0$ or $\nabla_T^2 \mathcal{H}_z + h^2 \mathcal{H}_z = 0$

where $h^2 = \gamma^2 + k^2$

and apply boundary conditions to find $h = h_1, h_2, h_3 \ldots$

These discrete allowed values of h are called the eigenvalues of the boundary value problem. Each value of h_n corresponds to a different "mode" of wave propagation. 30-13

Cutoff Frequencies

Corresponding to each value of h_n, we define the cutoff frequency of the nth mode, ω_{cn}; viz.,

$$\omega_{cn}^2 \mu \epsilon = h_n^2$$

Then the propagation const. γ is different for each mode and is given by

$$\gamma_n^2 = h_n^2 - k^2 = \omega_{cn}^2 \mu\epsilon - \omega^2 \mu\epsilon$$

If $\omega > \omega_{cn}$, $\gamma_n^2 = -\mu\epsilon\omega^2\left(1 - \frac{\omega_{cn}^2}{\omega^2}\right) = -\beta^2$

i.e. γ_n is imaginary and mode n can propagate

$$\beta = \sqrt{\mu\epsilon}\,\omega\sqrt{1 - \frac{f_{cn}^2}{f^2}}, \quad Z_{TM} = \frac{\gamma}{j\omega\epsilon} = \frac{j\beta}{j\omega\epsilon} = \eta\sqrt{1 - \frac{f_{cn}^2}{f^2}} < \eta$$

$$Z_{TE} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\beta} = \eta\frac{1}{\sqrt{1 - (f_{cn}^2/f^2)}} > \eta$$

30-14

Analysis of Rectangular Waveguides, TE Waves

TE Waves, $\mathcal{H}_z \neq 0$, $\mathcal{E}_z = 0$

Solve the scalar Helmholtz eq. in $\mathcal{H}_z$

$$\nabla^2 \mathcal{H}_z + k^2 \mathcal{H}_z = 0$$

Assume $\mathcal{H}_z = \mathcal{H}_z^0(x,y)\, e^{-\gamma z}$

Then $\nabla_T^2 \mathcal{H}_z^0 + (\gamma^2 + k^2)\mathcal{H}_z^0 = 0$

or $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + h^2\right)\mathcal{H}_z^0 = 0$

where $h^2 = \gamma^2 + k^2$

Method of Separation of Variables

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) \mathcal{H}_z^0 = 0$$

Try solving by assuming $\mathcal{H}_z^0 = X(x)\,Y(y)$

Then $\quad Y \dfrac{d^2X}{dx^2} + X \dfrac{d^2Y}{dy^2} + h^2 XY = 0$

or $\quad \dfrac{1}{X}\dfrac{d^2X}{dx^2} + \dfrac{1}{Y}\dfrac{d^2Y}{dy^2} + h^2 = 0$

This equation can be satisfied for all values of x and y iff

$$\frac{1}{X}\frac{d^2X}{dx^2} = \text{const.} \equiv -k_x^2 \quad \underline{\text{and}} \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = \text{const.} \equiv -k_y^2$$

Then $k_x^2 + k_y^2 = h^2 \equiv \omega_c^2 \mu\epsilon$

Solving the Ordinary Differential Equations

$$\frac{d^2X}{dx^2} + k_x^2 X = 0 \quad \text{and} \quad \frac{d^2Y}{dy^2} + k_y^2 Y = 0$$

Solutions are $X = A \sin k_x x + B \cos k_x x$

and $Y = C \sin k_y y + D \cos k_y y$

or $\mathcal{H}_z^\circ = XY = (A \sin k_x x + B \cos k_x x)(C \sin k_y y + D \cos k_y y)$

Find the transverse field components from the expression derived from Maxwell's Eqs. for TE waves:

$$\mathcal{H}_x^\circ = \frac{-1}{Z_{TE}} \mathcal{E}_y^\circ = -\frac{\gamma}{h^2} \frac{\partial \mathcal{H}_z^\circ}{\partial x}$$

$$\mathcal{H}_y^\circ = \frac{1}{Z_{TE}} \mathcal{E}_x^\circ = -\frac{\gamma}{h^2} \frac{\partial \mathcal{H}_z^\circ}{\partial y}, \quad \text{where } Z_{TE} = \frac{j\omega\mu}{\gamma}$$

33-4

Transverse Electric Field Components for TE Waves

$$\mathcal{E}_x^0 = -\frac{j\omega\mu}{h^2}\frac{\partial \mathcal{H}_z^0}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2}\left(A\sin k_x x + B\cos k_x x\right)\left(C k_y \cos k_y y - D k_y \sin k_y y\right)$$

$$\mathcal{E}_y^0 = \frac{j\omega\mu}{h^2}\frac{\partial \mathcal{H}_z^0}{\partial x}$$

$$= \frac{j\omega\mu}{h^2}\left(A k_x \cos k_x x - B k_x \sin k_x x\right)\left(C\sin k_y y + D\cos k_y y\right).$$

Boundary Conditions: Assume walls are perfect conductors.
Then $\mathcal{E}_{tan} = 0$ at all 4 walls

i.e. at $x=0$ and $x=a$, $\mathcal{E}_y^0 = 0 \Rightarrow A=0$, $k_x = m\pi/a$, $(m=0,1,2\ldots)$

and at $y=0$ and $y=b$, $\mathcal{E}_x^0 = 0 \Rightarrow C=0$, $k_y = n\pi/b$, $(n=0,1,2\ldots)$

Fields of the TE_{mn} modes

$$\mathcal{H}_z^o = B\cos\left(\frac{m\pi x}{a}\right) D\cos\left(\frac{n\pi y}{b}\right) \equiv H_o\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$\mathcal{E}_x^o = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right)H_o\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$\mathcal{E}_y^o = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_o\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$\mathcal{H}_x^o = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_o\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$\mathcal{H}_y^o = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_o\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi y}{b}\right)$$

and $k_x^2 + k_y^2 = h^2 = \omega_c^2\mu\epsilon$

$$\omega_{c,m,n}^2\,\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

The Dominant Mode

$$\omega_{c,mn}^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2, \quad a > b$$

Lowest value of $\omega_{c,mn}^2$ is obtained for $m=1$ and $n=0$; i.e. for the TE_{10} mode. This is the dominant mode in rectangular wvgd. (although we still need to check out TM modes)

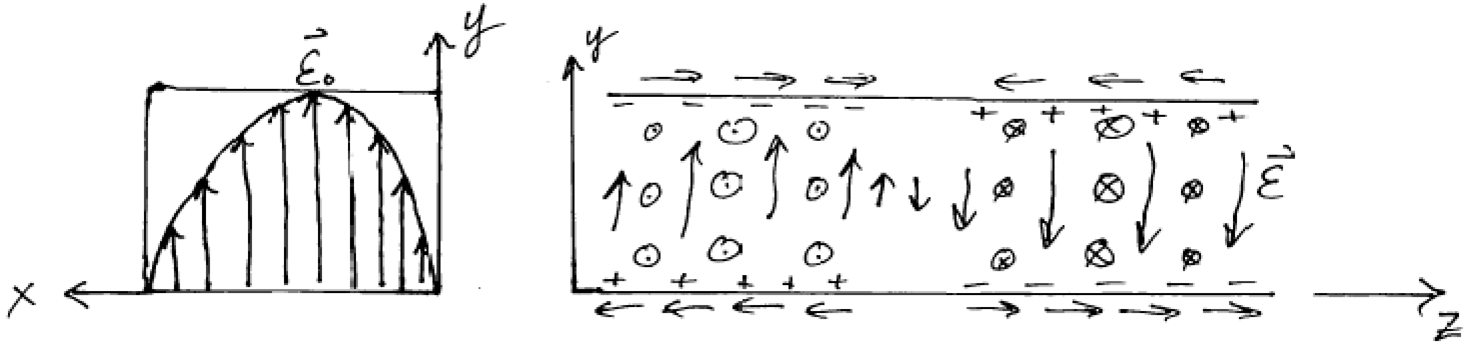
$$\omega_c(TE_{10}) = \frac{1}{\sqrt{\mu\epsilon}} \frac{\pi}{a}$$

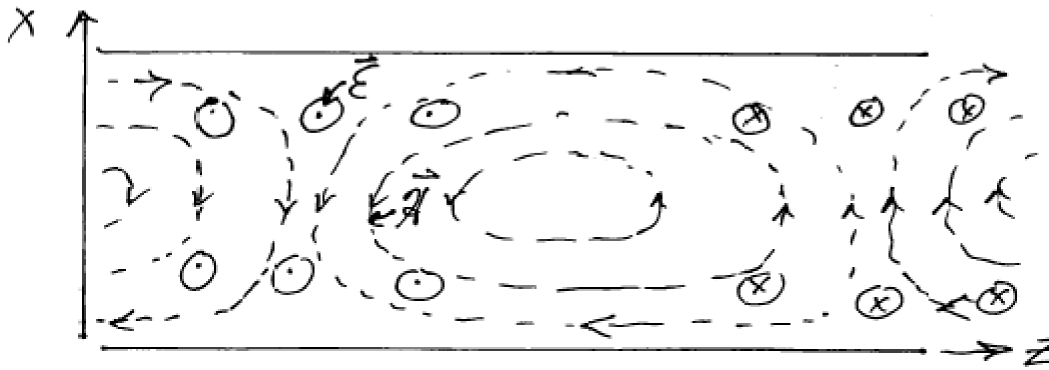
For the TE_{10} mode $\mathcal{E}_x^\circ = \mathcal{H}_y^\circ = 0$

$$\mathcal{E}_y^\circ = -\frac{j\omega\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) \equiv E_0 \sin\left(\frac{\pi x}{a}\right)$$

$$\mathcal{H}_x^\circ = \frac{j\beta}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) = -\frac{E_0}{Z_{TE}} \sin\left(\frac{\pi x}{a}\right)$$

Field Patterns for the $TE_{10}^{\sqcup}$ mode





$$\mathcal{E}_y = \mathcal{E}_y^0 e^{\gamma z} = \mathcal{E}_y^0 e^{j\beta z}$$

$$E_y = \text{Re}\left[\mathcal{E}_y e^{j\omega t}\right] = \frac{-\omega\mu}{h^2}\frac{\pi}{a} H_0 \sin\frac{\pi x}{a}\, \text{Re}\left[j\, e^{j(\omega t-\beta z)}\right]$$

$$= \frac{\omega\mu}{h^2}\frac{\pi}{a} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t-\beta z)$$

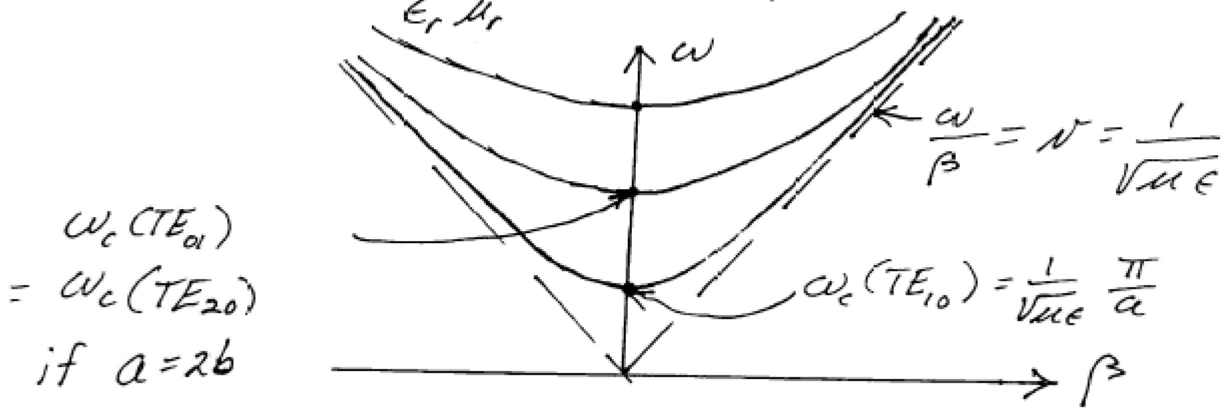
33-8

Dispersion Curves for TE modes

$$h^2 = \omega_c^2 \mu\epsilon = \gamma^2 + k^2 = \gamma^2 + \omega^2\mu\epsilon$$

$\gamma \cong j\beta$ if losses are negligible

$$\text{Then} \quad \omega^2 = \frac{\beta^2 c^2}{\epsilon_r \mu_r} + \omega_c^2 = \beta^2 v^2 + \omega_c^2$$



$$\omega_c(TE_{01}) = \frac{1}{\sqrt{\mu\epsilon}} \frac{\pi}{b} \quad , \quad \omega_c(TE_{20}) = \frac{1}{\sqrt{\mu\epsilon}} \frac{2\pi}{a}$$

To maximize freq. range of dominant mode + power capacity of wvgd. chose $b = \frac{a}{2}$

33-9

Rectangular Waveguide

Hollow waveguides are high-pass devices allowing e.m. wave propagation for frequencies above a cutoff frequency ($f > f_c$)

Propagation is in modes with well defined patterns of the e.m. fields (m peaks in magnitude across the wide dimension and n peaks across the small dimension) and with either an axial magnetic field (TM modes) or an axial electric field (TM modes)

For gas-filled waveguide with large dimension, a, and small dimension ,b, cutoff frequency for TE_{mn} or TM_{mn} modes is

$$f_c = 0.5\, c\, [\, (m/a)^2 + (n/b)^2\,]^{1/2}$$

Fundamental Mode (mode with lowesr cutoff freq.) is TE_{10} for which

$$f_c = 0.5c/a$$

Usually a = 2b, and then the fundamental mode is the only propagating mode over an octave in frequency (factor of 2)

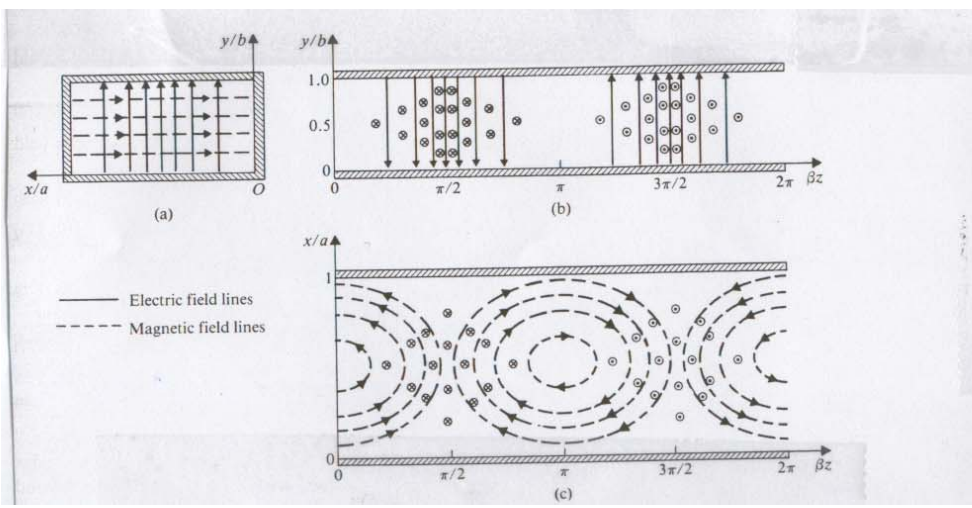
E.M. Fields in the TE_{10} Mode

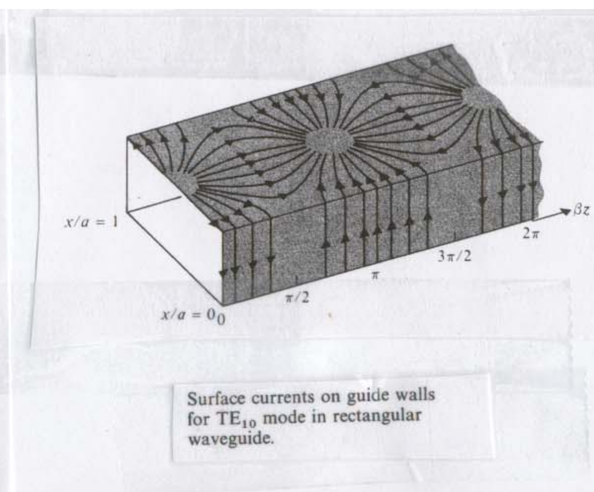
$$E_y = E_o \sin(\pi x/a) \sin(\omega t-\beta z)$$

$$H_x = -(Z_{TE})^{-1} E_y, \quad \text{where } Z_{TE} = (\mu/\varepsilon)^{1/2}[1-(f/f_c)^2]^{-1}$$

$$H_z = (2f\mu a)^{-1} E_o \cos(\pi x/a) \cos(\omega t-\beta z)$$

$$E_z = E_x = H_y = 0$$





Surface currents on guide walls for TE_{10} mode in rectangular waveguide.

Attenuation in the TE_{10} Mode

$$(\alpha_c)_{TE_{10}} = \frac{R_s[1 + (2b/a)(f_c/f)^2]}{\eta b\sqrt{1-(f_c/f)^2}}$$

$$= \frac{1}{\eta b}\sqrt{\frac{\pi f \mu_c}{\sigma_c[1-(f_c/f)^2]}}\left[1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right] \quad \text{(Np/m)}.$$

1 Neper/m = 8.69 dB/m

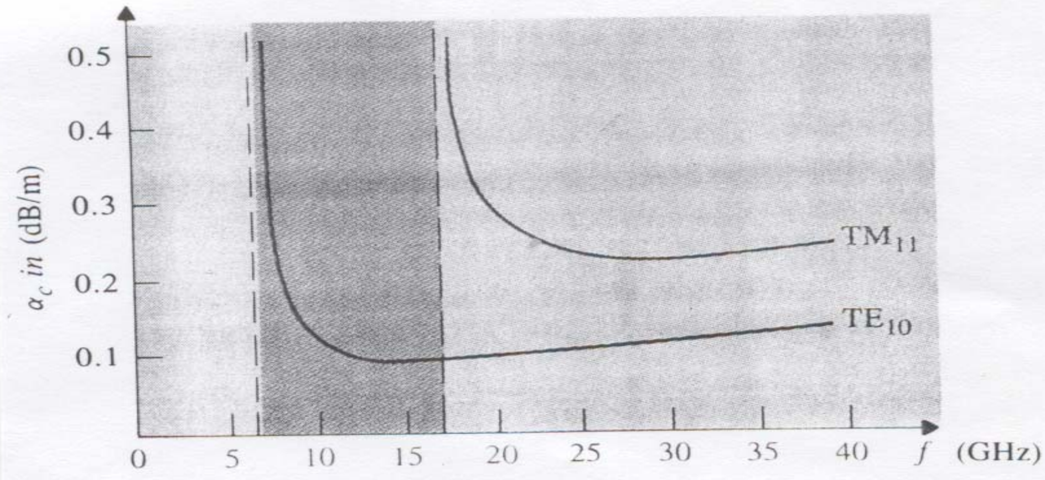


FIGURE 10–14
Attenuation due to wall losses in rectangular copper waveguide for TE_{10} and TM_{11} modes. $a = 2.29$ (cm), $b = 1.02$ (cm).

Specs of L-Band Rectangular Waveguide

Waveguide Designation: WR 770 or RG-205/U

Inner Dimensions: 7.7 inches x 3.85 inches
19.56 cm x 9.78 cm

Cutoff Freq. (TE_{10}): 0.767 GHz

Recommended Freq. Range: 0.96 – 1.45 GHz

Attenuation: 0.201 –0.136 dB/100ft.
0.066 – 0.0045 dB/m

Power Rating of Rectangular Waveguide

Power flow in fundamental mode in rectangular waveguide

$$P = 0.25\, E_o^2\, a\, b\, [\, (\omega/c)2 - (\pi/a)2\,]^{1/2}$$

Air at STP breaks down when E = 3 MV/m (dc value)
Set E_o= 3 MV/m to find maximum power flow; this will allow some safety margin since breakdown field for microwave pulses will be higher than dc value.

Sample calculation: f = 109 Hz, a = 0.1956 m, b = 0.5 a,

P_{max} = 73 Megawatts

This may be increased by a factor of $(3p)^2$ if the air in the waveguide is replaced by pressurized SF_6 at a pressure of p atmospheres; i.e. with 1 atm. of SF_6, P_{max} = 657 MW while at a pressure of 2 atm., P_{max} = 2.6 GW.

Circular Waveguide

Fields vary radially as Bessel functions.

Fundamental mode is TE_{11} with $f_c = 0.293\ c/a$

Next lowest cutoff freq. is for the TM_{01} mode with $f_c = 0.383\ c/a$

Range for single mode operation is smaller than an octave.

571

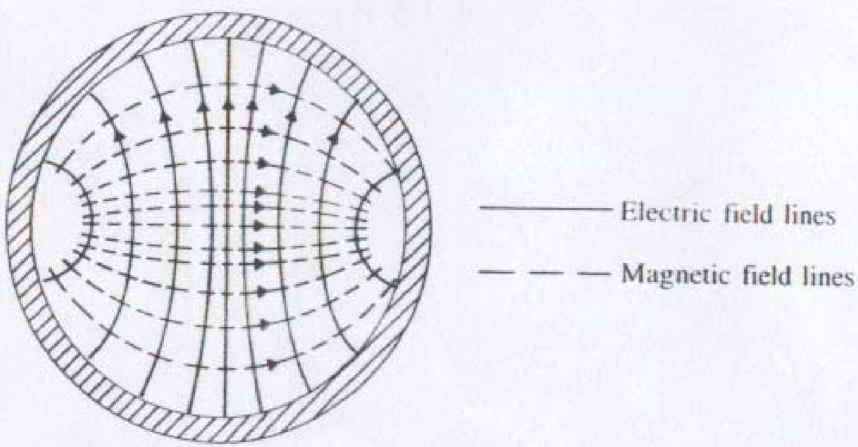


FIGURE 10-21
Field lines for TE_{11} mode in a transverse plane of a circular waveguide.

Antenna Fundamentals

An antenna may be used either for transmitting or for receiving microwave power. When used for receiving, the antenna is characterized by an effective area,

$$A_e = \text{power received /power density at the antenna}$$

When used for transmitting the same antenna is characterized by its gain, G, which is related to its effective area by the universal relationship

$$G = (4\pi/\lambda^2)\,A_e$$

For an aperture antenna such as a waveguide horn or a parabolic reflectorwith physical aperture area, $A_{phys,}$

$$A_e = K\,A_{phys}$$

where K is an efficiency factor <1 to account for nonuniformity of the field in the aperture, Ohmic losses, in the antenna walls, etc.

Beamwidth

Antenna gain, G, is the ratio of the maximum power density achieved in a preferred direction with the aid of the antenna compared with the power density that would be achieved with an isotropic radiator; i.e.

$$G = S_{max} / S_I \quad \text{where } S_I = (P_t / L_t)\, 4\pi R^2$$

where (P_t/L_t) is the total power fed to the antenna.

Gain is achieved by concentrating the electromagnetic radiation into a beam whose width is inversely related to G

We have the relationship

$$G = K\, 4\pi / [(\Delta\phi)^{rad} \; (\Delta\theta)^{rad}\,] = K\, 41{,}000 / [\, (\Delta\phi)^{o}\, (\Delta\theta)^{o}\,]$$

where for a wave propagating in the radial direction in spherical coordinates, $\Delta\phi$ and $\Delta\theta$ are the 3 dB beamwidths respectively in the azimuthal and polar directions and the superscripts indicate measurement in radians or degrees.

Parabolic Dishes

Gain = k $(\pi D)^2 / \lambda^2$

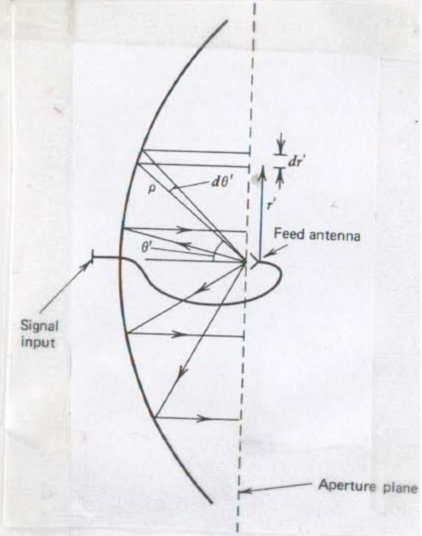
Where k = efficiency factor, ~55%

G = gain over isotropic

D = dish diameter

λ = wavelength

Beamwidth = $70^\circ \lambda / D$



Efficiency factor has an ideal value when only field non-uniformity in the aperture plane is taken into account of K = 55%. Empirically, K ~40%

Horn Antennas

Gain = K 4π A B / λ^2

K = efficiency factor ~ 80%

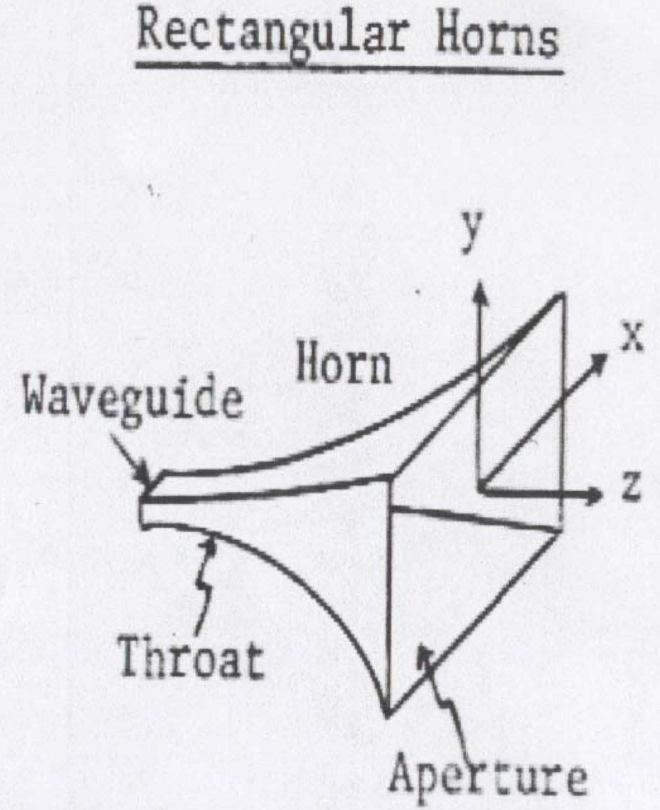
Half-power Beam Widths

E plane (yz)

53° / (B / λ)

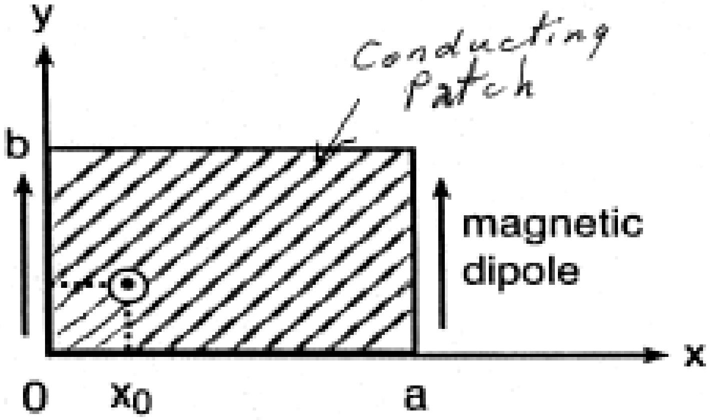
H plane (xz)

80° / (A / λ)

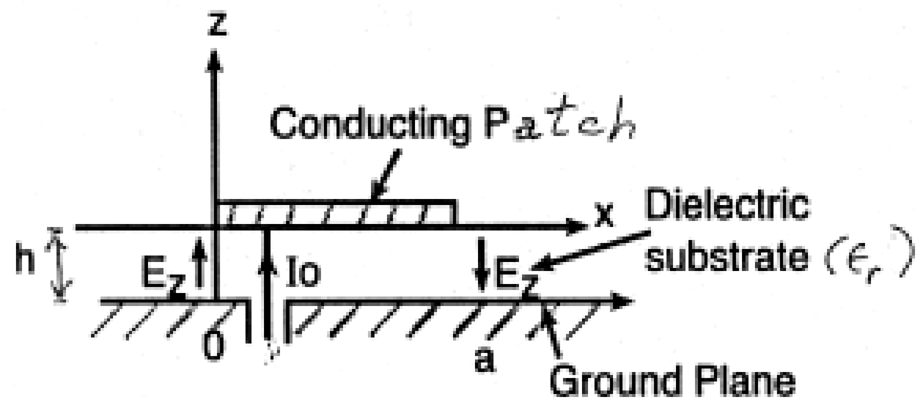


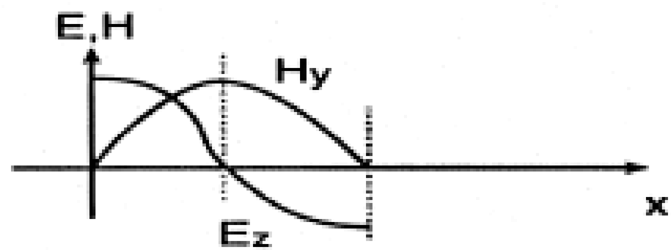
Ideally, K = 80%. Empirically K ~ 50%.

Microstrip Patch Antennas (Array of microstrip patches used in ADT non-lethal weapon system)



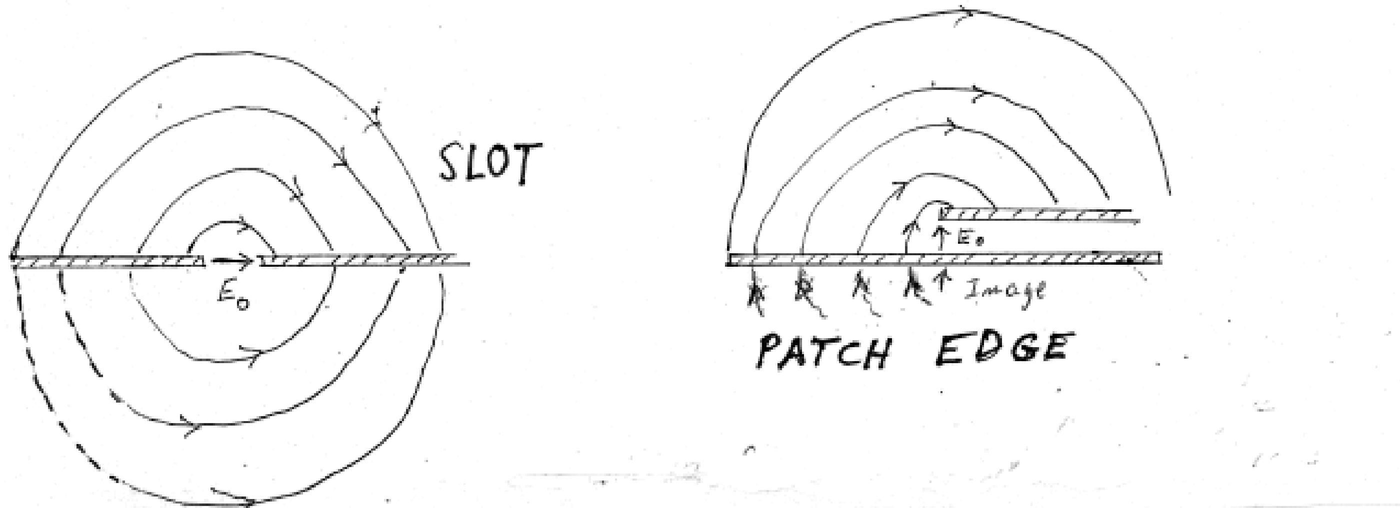
TM$_{10}$ mode excited in cavity with open ckt. boundaries





E$_z$ field along edges at X=0 and X=a looks like magnetic equivalent of dipole antennas which are in-phase.

Radiation from Patch Edge vs, Radiation from a slot



Radiation from patch edge is like radiation from a slot, but it is only into a half space. However, with image on ground plane, there are equivalently 2 antennas radiating into the half space.

If the slot were λ/2 long it would have an antenna gain of G = 1.64

The patch edge radiating into a half-space has gain G = 4 x 1.64 = 6.5

(Losses have been neglected)

Microstrip Patch Antenna Array

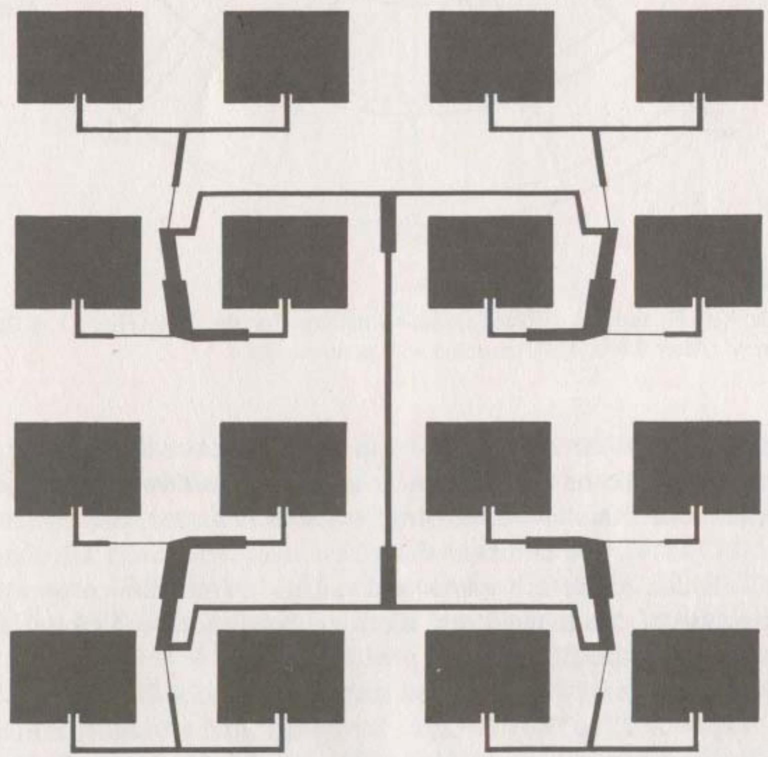


FIGURE 9.89. 4 × 4 element reduced side-lobe array. (After Weiss [119] protected by U.S. Patent Numbers Re 29,296 and 29,911, reprinted with permission.)

Broadside array gain
= 6.5 x No. patch edges
Since each patch has 2 radiating edges gain
G = 6.5 N K
Where N is the no. of Patches and K is an efficiency factor

Characteristics of Microstrip Patch Antennas

- $Q \sim 30$. Bandwidth small (~3%)
- Poor endfire radiation characteristics

↑ disadvantages

↓ advantages

- Ease of construction
- Low cost
- Compact, low profile
- Can be wrapped around cylinder (aircraft fuselage)
- High gain from large planar array

Microstrip Array Antenna Example

Antenna dimensions: 1.5 m x 1.5 m

Wavelength: 3 mm

Length of patch edge: 1.5 mm

Spacing between patches:1.5 mm

(in direction of radiating edges)

No. of patches, N: $[1.5/(3 \times 10^{-3})]^2 = 2.5 \times 10^5$

(assuming same no. of patches in each direction)

Gain, $G = 6.5 \times N K = 6.4 \times 10^5$ or ~58 dBi

(assuming K ~ 40%)

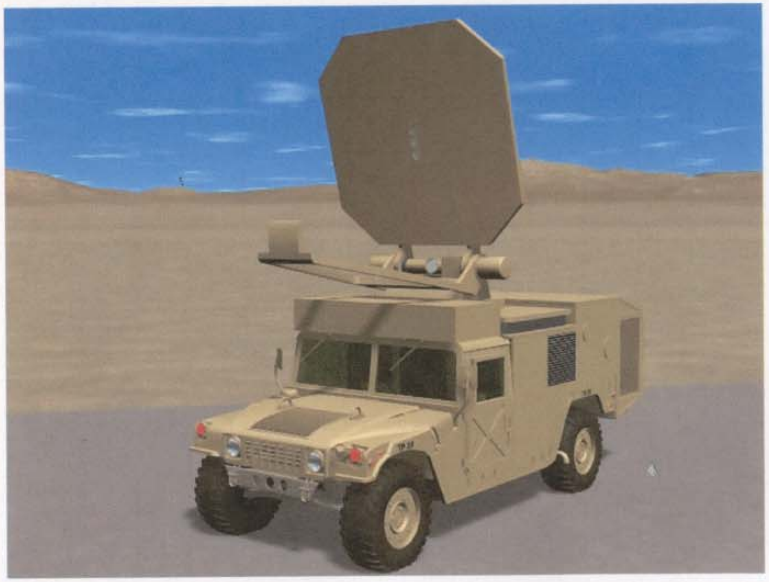
This value of gain is of the same order as gain for a parabolic reflector with the same area;

viz., $0.4 \times 4\pi A/\lambda^2 = 1.3 \times 10^6$ or ~61 dBi

Technology/System Description

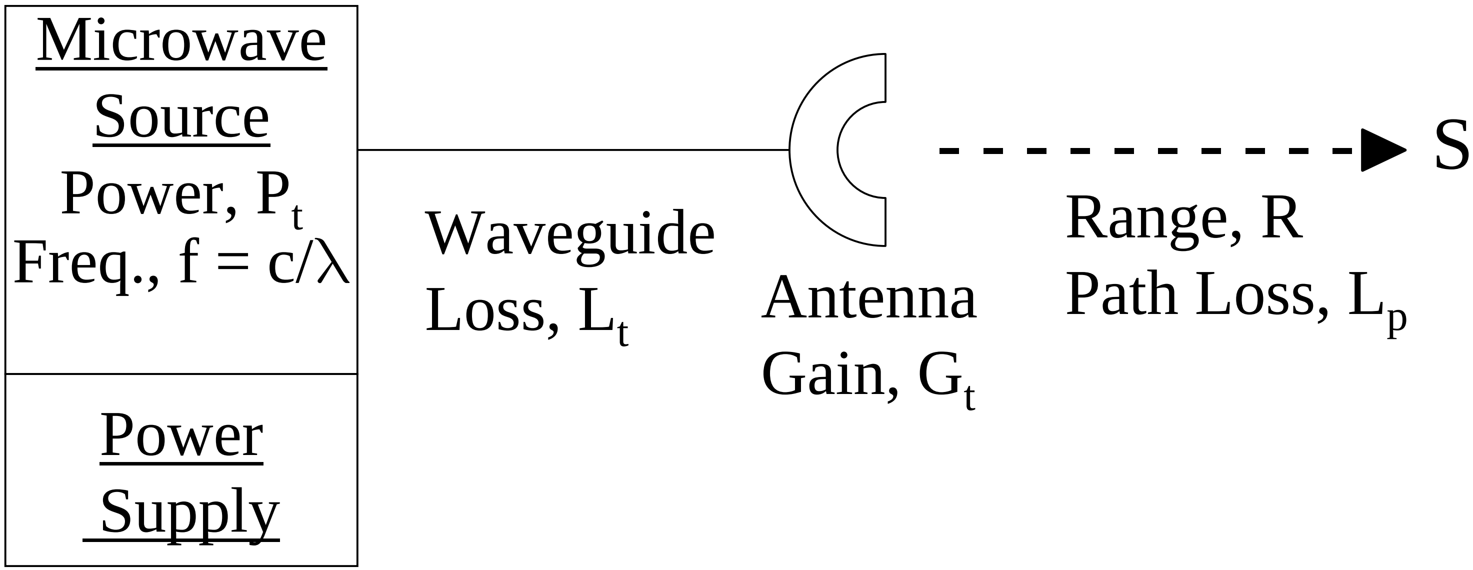
Name: Active Denial Technology

Description:
- ***Nonlethal antipersonnel directed energy weapon***
- ***Long range, lightspeed, line-of-sight, deep magazine***
- ***Energy beam creates a directional sensation of pain, causing repel without damage***



Potential Applications:
- ***Area Delay/Denial***
- ***Force Protection***

HPM Sources, Waveguides, Antennas and Propagation



Effective Isotropic Radiated Power, EIRP = Pt Gt / Lt

Power Density at "Target", $S = (4\pi/\lambda^2)$ (EIRP/ L_p)

In free space, $L_p = L_f = (4\pi R / \lambda)^2$ and $S = \text{EIRP} / (4\pi R^2)$

Power Density Delivered by ADT

If we assume that path loss is its free space value

$$S = EIRP / (4\pi R^2)$$

Transmitter power = 100 kW
Estimated antenna gain = 61 dBi
Estimated feeder waveguide losses = 4 dB
Then, EIRP = 5×10^{10} Watts

At a range of R = 1 km, S = 0.4 Watts/cm^2

Note: To extend range new gyromonotron sources are being developed with power of 1 MW and higher

Non-Developmental RF Threat Demo

{Filmed/Live at NSWCDD, VA in 1998}

Demonstrate *TRUE* capabilities of non-developmental RF threats to equipment [Requested by OSD(C3I)]

ARL L-Band Source:
- 2 MW peak power
- 2 μs pulse width
- 300 Hz repetition
- 13 dB gain horn
- 1.3 GHz

NSWCDD Bow-Tie Antenna
- 150 kV Marx generator

NSWCDD Discone Antenna
- 300 kV Marx generator

Power Density Delivered by ARL L-Band Source

Transmitter Power: 2 MW
Antenna Gain: 13 dBi
Feeder Wavegide Loss: assumed negligible
EIRP = 4 x 10^6 Watts

At a range, R = 10 meters
and assuming free space propagation loss

$S = EIRP / (4\pi R^2) = 0.32$ Watts/cm^2

Extra loss when Penetrating into Buildings

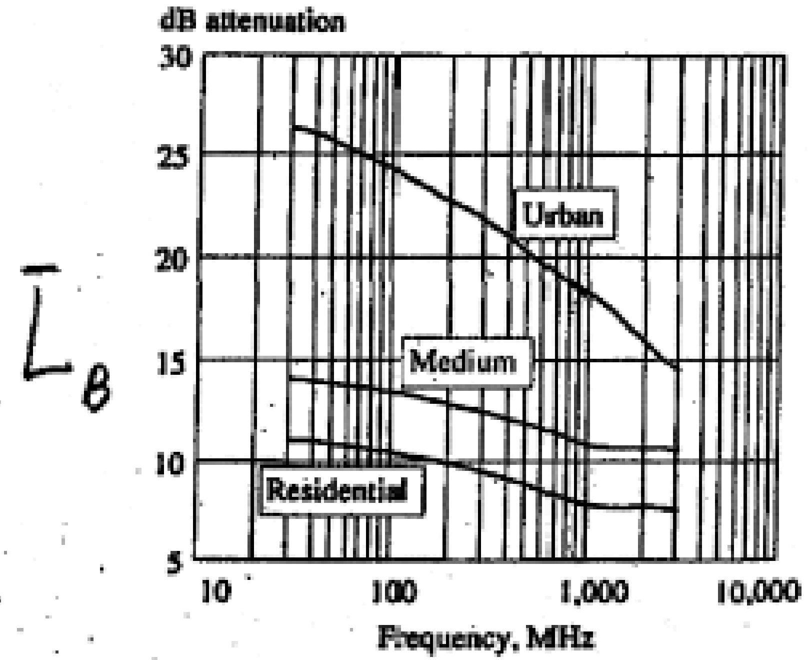


Figure 7.16 Building attenuation at ground level.

Table 7.5
Building Type Definitions

Building Type	Description
Urban	Typically large downtown office and commercial buildings, including enclosed shopping malls
Medium	Medium-size office buildings, factories, and small apartment buildings
Residential	One- and two-level residential buildings, small commercial and office buildings

Power Density from an Explosively-Driven HPM Source above a Building

Estimated Transmitter Power: 2 GW centered at 1 GHz

Esimated Antenna Gain: 1

Estimated Feeder Loss: 1

EIRP = 2×10^9 Watts

Estimated Range: R = 25 m

Free Space Path Loss: $L_f = (4\pi R / \lambda)^2 = 1.1 \times 10^6$

Building Loss: $L_B(dB) = 18dB$ or $L_B = 63$

Total Path Loss: $L_P = L_f L_B = 6.9 \times 10^7$

Power Density at Site of Electronics Inside Building:

$$S = (4\pi / \lambda)^2 (EIRP / L_P) = 5.1 \text{ Watts / cm}^2$$

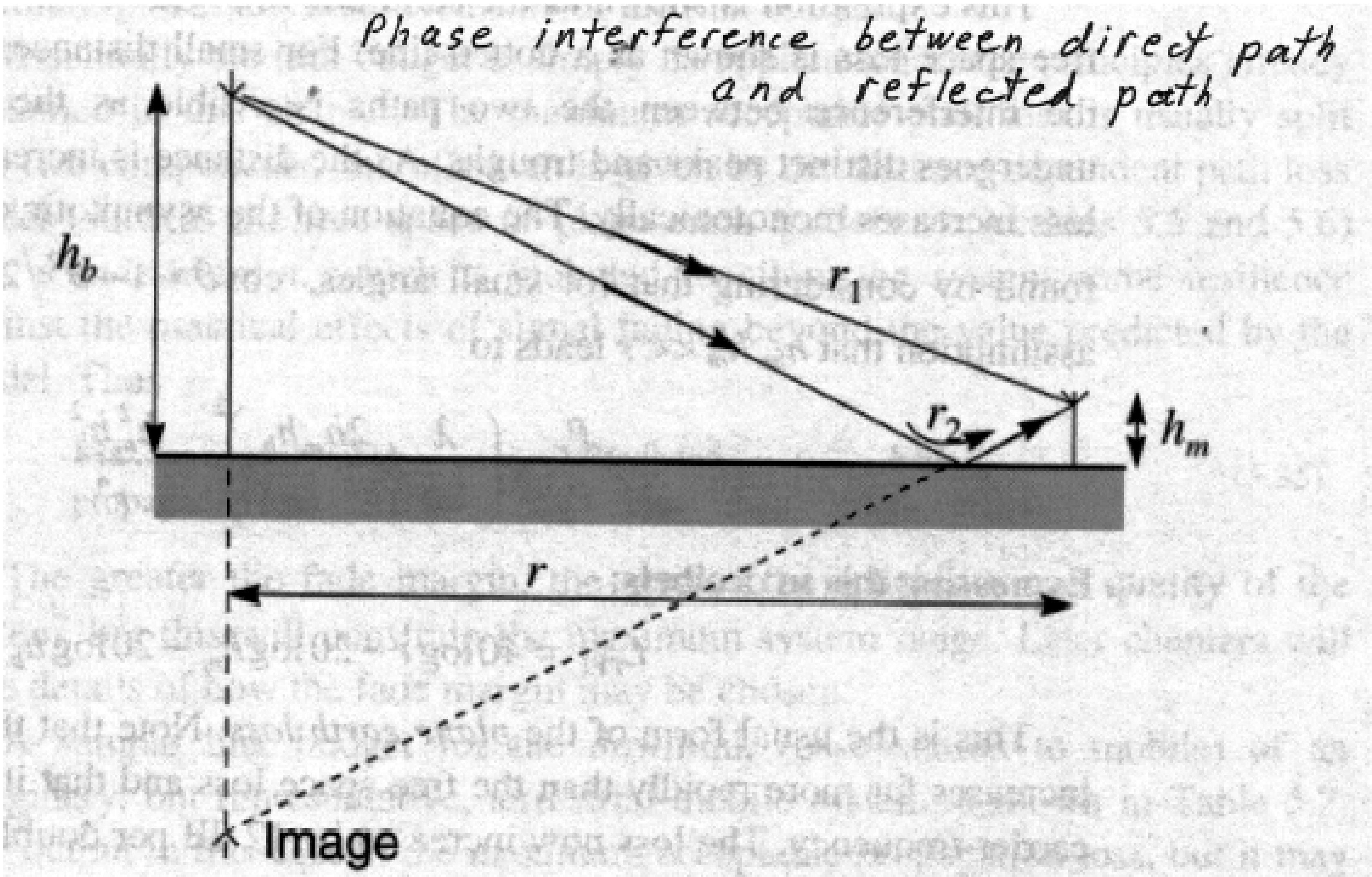
Other Factors Contributing to Propagation Loss

We saw that path loss may be significantly increased by the reflection, refraction and absorption that occur when the microwaves pass through a wall.

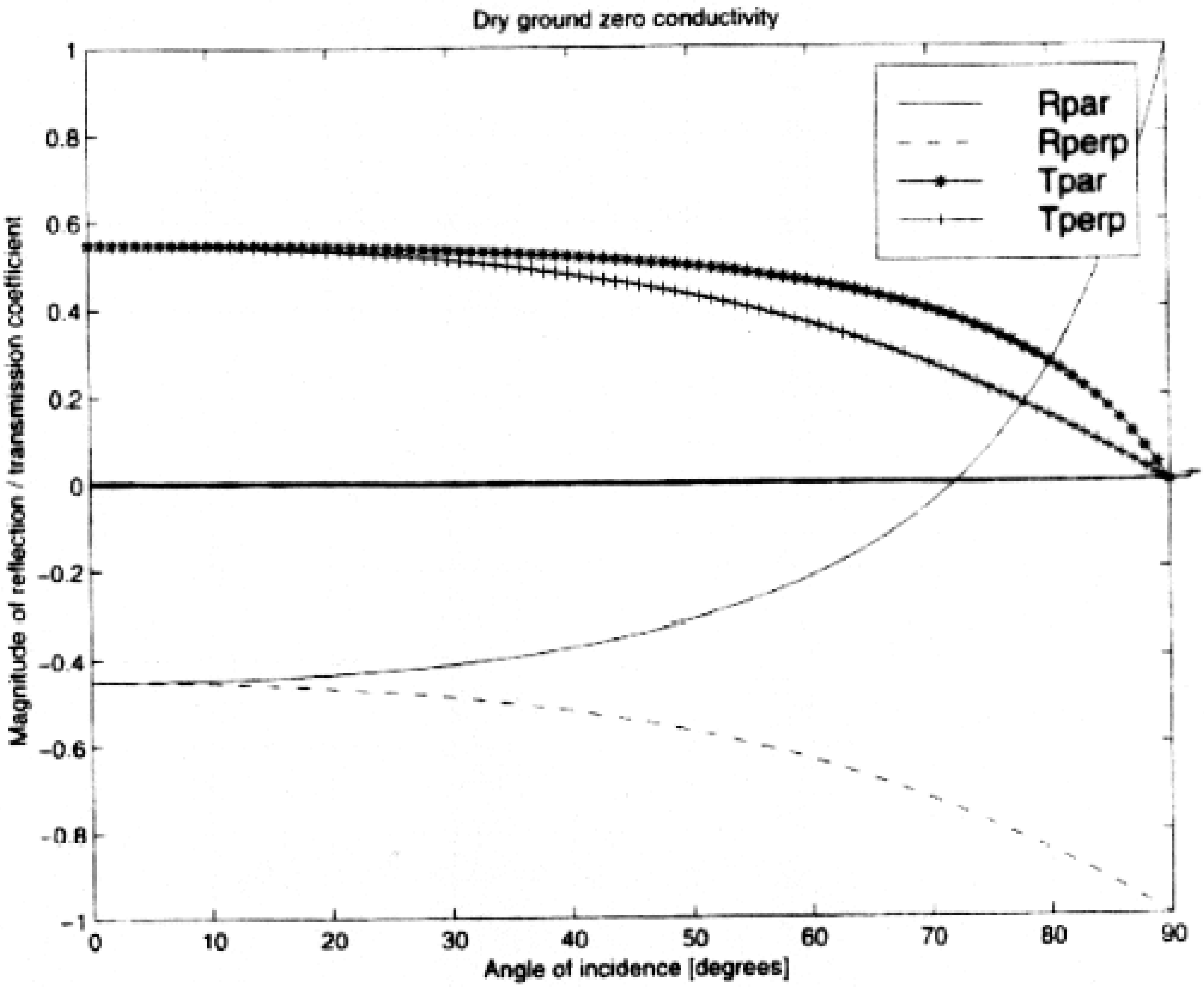
Reflection can also result in multiple paths for the microwave propagation and extra loss because of multi-path interference

Obstructions near the wave path can diffract the wave and cause additional losses.

Plane Earth Path Loss: A Case of Multi-Path Interference



Reflection and Transmission Coefficients



Plane Earth Path Loss

$$r_1 = \sqrt{(h_b - h_m)^2 + r^2}\,, \quad r_2 = \sqrt{(h_b + h_m)^2 + r^2}\,, \quad r_2 - r_1 \approx \frac{2 h_m h_b}{r}$$

Amplitude of electric field at receiving antenna

$$A_{total} = A_{direct} + A_{refl.} = A_{direct}\left|1 + R\exp\left(-j\frac{k2h_bh_m}{r}\right)\right|$$

Now, if $\frac{2kh_bh_m}{r} \ll 1$, $\exp\left(-j\frac{2kh_bh_m}{r}\right) \approx 1 - j\frac{2kh_bh_m}{r}$

and with grazing incidence + perp. pol., $R \approx -1$

Then $$A_{total} \approx A_{direct}\,\frac{2kh_bh_m}{r}$$

$$\therefore\ P_{s,total} \approx P_{s,direct}\,\frac{(2k)^2h_b^2h_m^2}{r^2} = \frac{P_tG_tG_r}{L_tL_r}\,\frac{(2k)^2h_b^2h_m^2}{L_F\,r^2}$$

$$\therefore\ \text{Path Loss},\ L_{p.e.} = L_F\frac{r^2}{(2k)^2h_b^2h_m^2} = \left(\frac{4\pi rf}{c}\right)^2\frac{r^2c^2}{(4\pi f)^2h_b^2h_m^2}$$

$$\text{or}\quad L_{p.e.} = \frac{r^4}{h_b^2h_m^2}$$

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Extra Path Loss Due to Diffraction

Cell phone path loss in an urban enviroment

$$L_{empirical} \sim \frac{r^4 f^2}{h_b^2 h_m}$$

Free space path loss

$$L_F \sim r^2 f^2$$

Plane earth path loss

$$L_{p.e.} = \frac{r^4}{h_b^2 h_m^2}$$

Physical model of cell phone path loss must involve diffraction (bending of waves around obstructions)

Cell Phone Propagation Loss in Urban Settings

A very useful formula for calculating path loss based upon a large number of measurements in U.S. cities is due to Egli. For $h_m < 10m$,

$$L_{Egli}(dB) = 40 \log R_{km} + 20 \log f_{MHz} - 20 \log h_b + 76.3 - 10 \log h_m$$

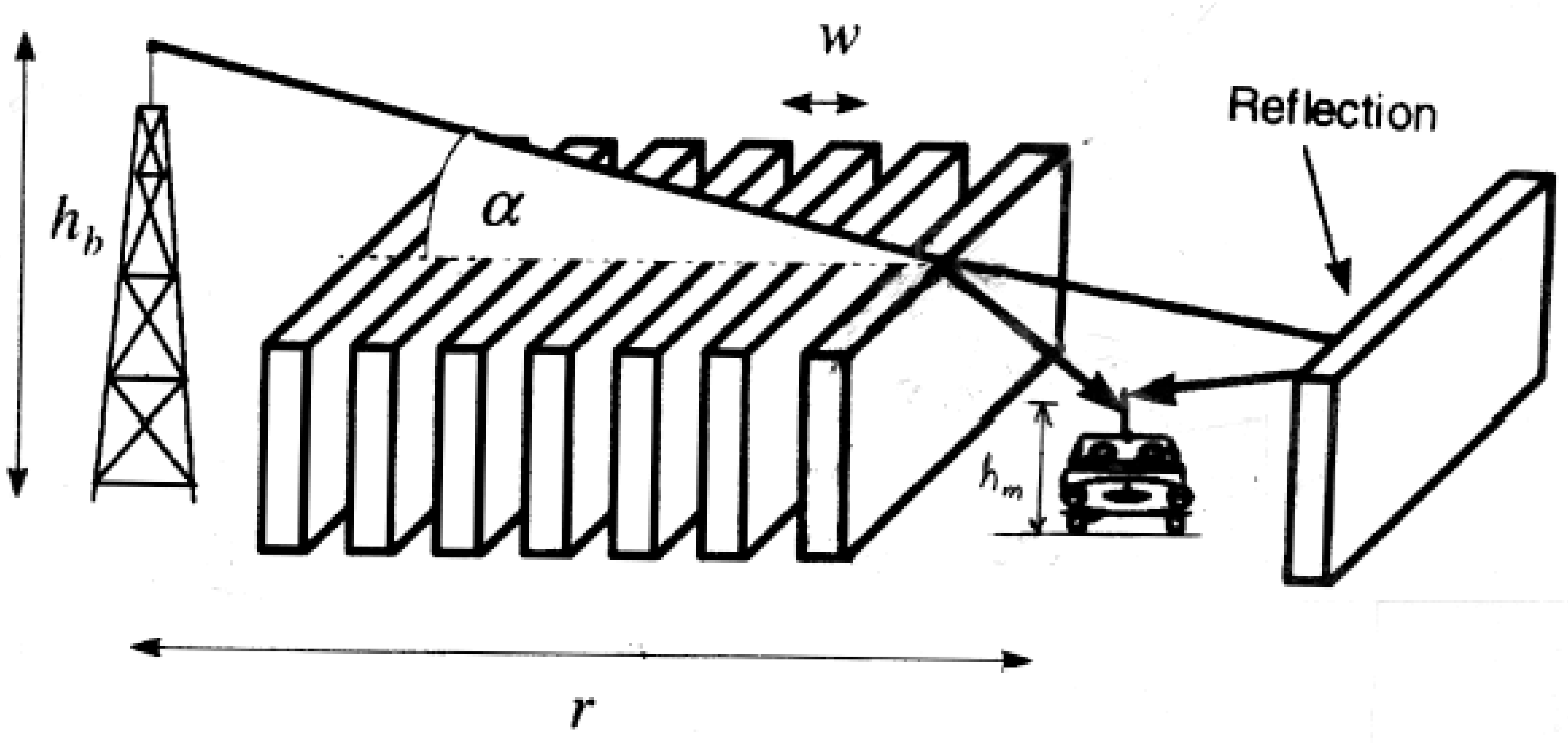
which may also be written as

$$L_{Egli} = 4.27 \times 10^{-17} \frac{r^4 f^2}{h_b^2 h_m}$$

which corresponds to the emprical dependence of path loss on the parameters presented previously

What physical model gives these dependences?

Geometry for Evaluating Cell Phone Path Loss



Knife Edge Diffraction

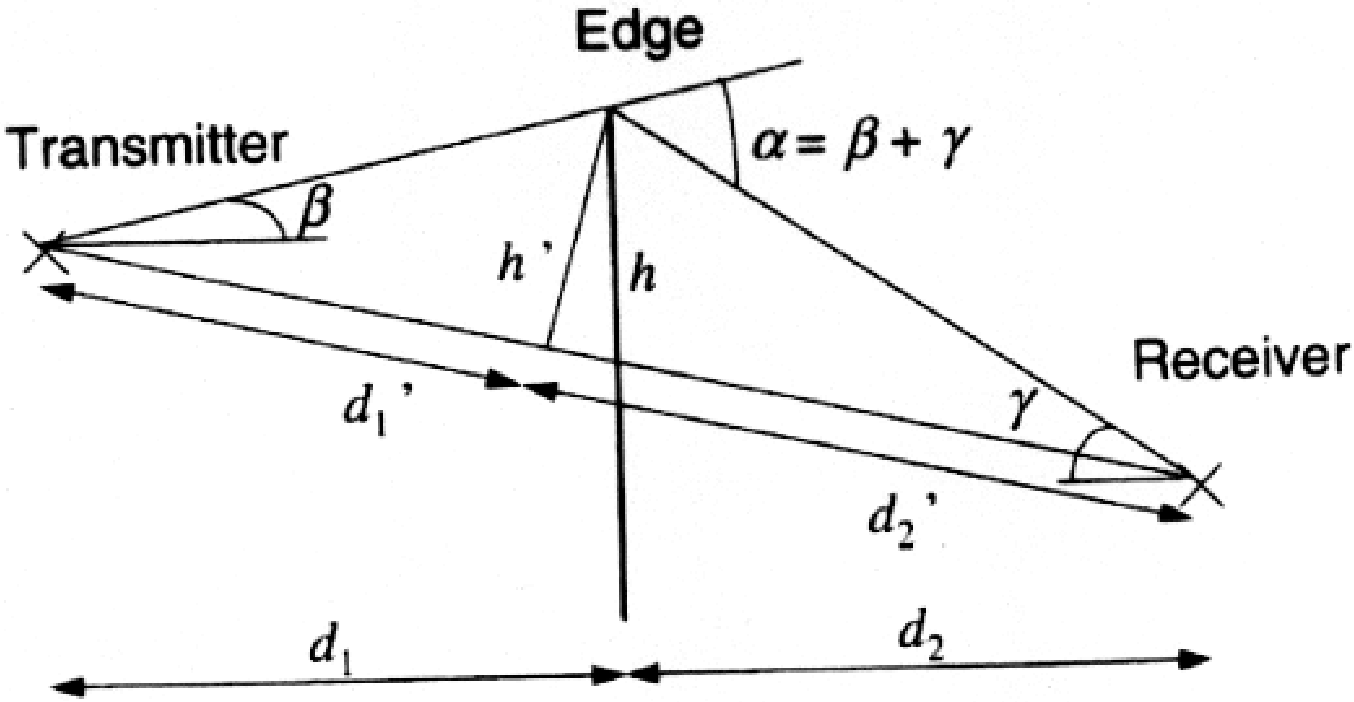


Figure 3.16: Knife-edge diffraction parameters

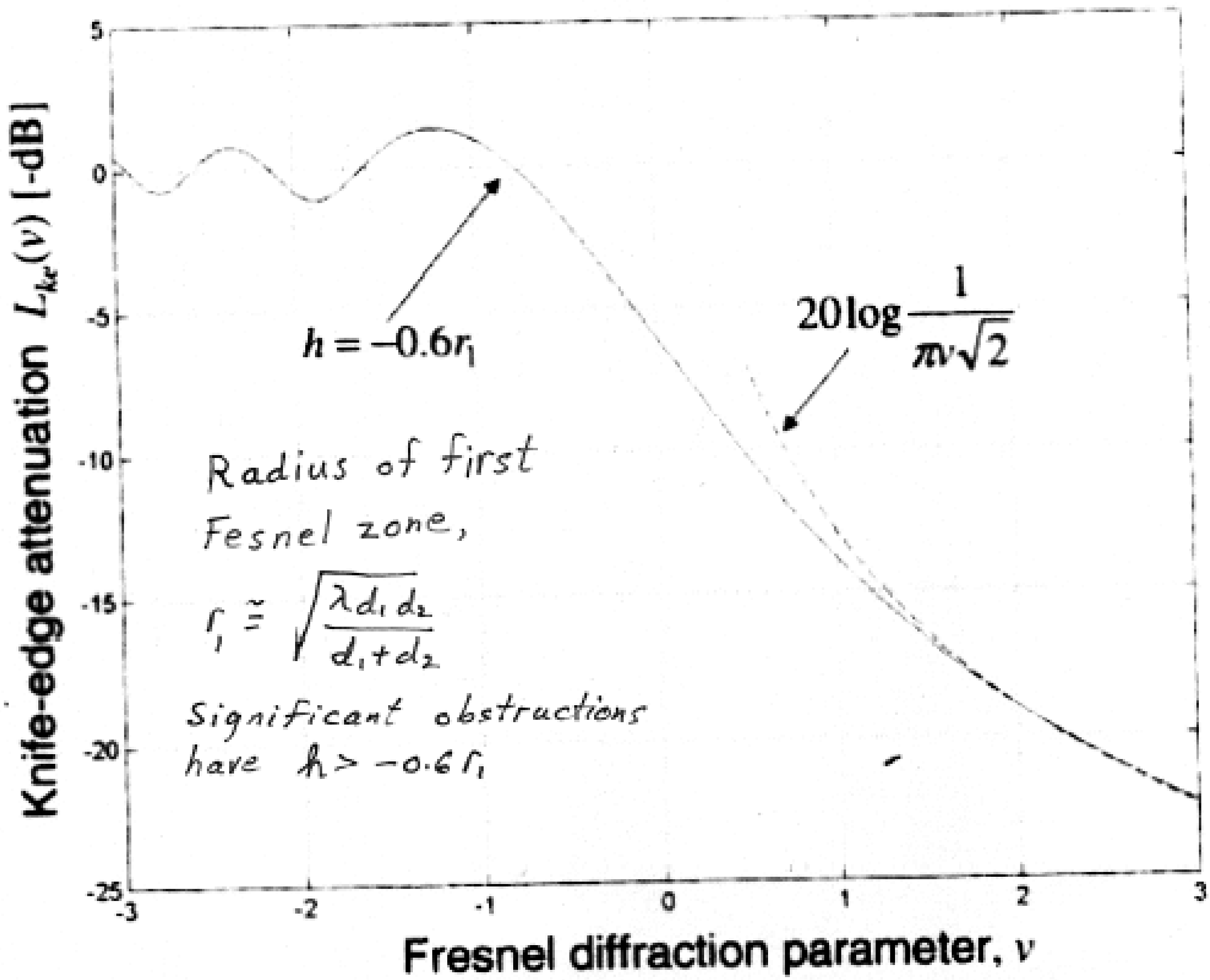
Diffraction Parameter

$$\nu = h' \sqrt{\frac{2(d_1' + d_2')}{\lambda d_1' d_2'}}$$

$$\approx h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}}$$

Attenuation by Knife Edge Diffraction

Note that there is attenuation even when ray path is above knife edge ($\nu < 0$)



3.15: Knife-edge diffraction attenuation: (—) exact (– –) large ν approximati

The Bottom Line on Propagation Loss

Make sure you are using an appropriate physical model of the propagation path!

Dependence of loss on range, frequency, antenna height and target height are strongly influenced by the physical processes along the propagation path (reflection, refraction, absorption, diffraction, multipath interference)