

## SLIDING CORRELATIONS, DELAY BASED DISCRIMINATORS , PROCESSING GAIN and their APPLICATION in a GPS RECEIVER

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This article is meant to introduce the reader to the principles Sliding Correlators and how they are used to create a delay discriminator. Additionally the spread spectrum term “Processing Gain” is discussed. There are many other types of correlators but here we will focus on the Sliding Type. Before we dive right in on the sliding correlator we need some intro material on the Correlation operation.

One of the key operations that distinguish the GPS receiver from classic narrow band receivers is the use of a correlator. The correlation process in the GPS receiver is used to align the replica C/A code with the transmitted C/A code. Additionally this results in recovery of timing signals that are ultimately used in the receiver TOA measurement process. But what is correlation and how does it work?

Fundamentally correlation is a statistical process, that is it is related to averages and probabilities. We intuitively know that when we roll a pair of dice the outcome from one throw to another is *not correlated*, that is the previous throws have no effect on subsequent throws. When two events are in some way interrelated such that the outcome of one affects the other we could say the events had some sort of correlation.

There is another interpretation of correlation and that is as a measure of *similarity*. Particularly in electronics it would be desirable to compare various time signals to one another and see if they have anything in common. By having such a tool it should be possible to quantitatively determine *how much and where* two signals are correlated and where *they are not* correlated. It is this interpretation of correlation that is used in the GPS receiver correlation process.

Finding the point in time where two signals are similar, or in GPS receiver where the receiver's replica C/A code is lined up with the received C/A code from the satellite, is the prime reason we need to understand the correlation process. We will see that correlation allows us to determine, to a very high degree of accuracy, when we have C/A code alignment.

Before we discuss correlation in more depth we need to briefly examine a concept called “*Time Average Value*” of a time varying voltage or current signal.

### Time Averaging

What is the “time averaging value” function? Let's look at the waveforms in figure A1. Figure A1-A shows a sine wave that has a max value of +1 volts and a minimum value of -1 volts. Its time average is zero volts. The reason is that the waveform spends as much time positive as it does negative and the magnitude of these positive and negative excursions is identical. The waveform shown in figure A1-B is a square wave that goes from 0 to 1 volt. The average value would be 0.5 volts. Waveform A1-C is a sine wave that goes from zero volts to 5 volts maximum. The average value of this waveform is 2.5 volts. Waveform A1-D is a small segment of random binary bit sequence with values +1 or -1. If the sequence is truly “random” and of large length the average value will be very close to zero. It should be apparent that the time average function of a waveform is the *DC value* of the waveform. If one had a perfect DC voltmeter (reads true independent of waveform type) and applied the waveforms of Figure A1 to it would read the Average Value. In lieu of the DC meter an analog Power meter can be used. The total power *minus* the AC coupled power is equivalent to measuring the Average or DC Value (if properly scaled).

After studying various waveforms and the time average function the reader should be able to draw a line on the waveform indicating the approximate average or DC value. It is hoped the reader will develop an intuitive feel for the time average function and be able to approximate this value for most common waveforms.

### Correlation, the Mathematical Statement

For those readers familiar with convolution, correlation is a closely related operation. Convolution and Correlation both use shifting, multiplying and integration operations on time waveforms (typically). This

discussion is restricted to time waveforms only. The mathematical formula for correlation of two time signals is;

$$z(t) = \int x(\tau) * y(t + \tau) d\tau \quad \tau \text{ covers } -\infty \text{ to } +\infty \quad \text{EQ(1A)}$$

Where  $x(t)$  and  $y(t)$  are time waveforms for the purposes of this discussion. The variable  $\tau$  is the time shift applied to  $y(t)$  and the variable of integration.  $z(t)$  is correlation waveform that we seek.

As usual writing such equations down does little to inform our intuition on what is really happening! But the math is the exact model that we will attempt to execute in the imperfect world of electronic circuits.

We need to break down EQ(1A) in to its individual operations so that we can better understand what this correlation process does and how we can implement it. First of all  $x(t)$  and  $y(t)$  are usually voltage waveforms of the sort you can see with an oscilloscope. If  $x(t)$  and  $y(t)$  are different waveforms the correlation of the two is sometimes called *Cross Correlation*. If they are the same waveform then the correlation process is a special case called *Auto-correlation*. In the real world of analog electronics it is almost impossible to have two waveforms be EXACTLY identical, but we can get very close. Auto-correlation is the function we are after for use in the GPS receiver.

Now lets examine the operations needed be EQ(1A);

- 1) Shift  $y(t)$  by  $\tau$  seconds
- 2) Multiply  $x(\tau)$  and  $y(t+\tau)$  together
- 3) Now find the average value of the resulting waveform, this is the integration process.

So our circuit will need a method to shift a time waveform with respect to another, a way to multiply them together and way to time average the result. Of these three operations the hardest to implement in circuitry is the shifting function. So let's leave that to last. First lets look at the multiply and averaging functions.

### **A Multiplier and Integrator for Digital signals**

The GPS receiver uses pseudo random sequences, which are digital signals that are encoded onto the RF carrier. They are digital in that they can only be -1 or +1, where here it is better to use a bipolar logic state. We need a multiplier that can take these two digital signals and multiply them together. Such a device is common in the digital logic family, it's just a Exclusive OR gate. We are going to use a modified , bipolar form of the EXOR gate where in/outs are +1 or -1. If you let the zero state of the standard EX OR function be -1 instead of zero you can see that this logic function does do a multiplier type function with the output inverted. We chose to ignore the inversion as it is of no consequence and call this model a BI-Polar EXOR (or EX-NOR more properly). The output of the multiplication is +1 or -1 as expected. The truth table is shown in Figure A2. So now we have our multiplier for our two digital signals, it is just an EX OR gate. The choice of special logic levels make discussion and modeling easier for using the EXOR function as a multiplier. From a practical point of view once signals are scaled properly and inversions addressed the standard EXOR logic gate is equivalent of the defined BI-Polar XOR.

We can approximate the time average function (or integration) with a simple Resistor/Capacitor lowpass filter on the output of the EX OR gate. The lowpass filter does not do exact integration but for our purposes its close enough. The choice of the time constant of this filter will depend on the code rate, code length and the rate, which the two signals are sliding by each other. We will come back to this later.

Figure A2 shows our simple digital correlator. We can supply the two time signals as digital signals to the inputs of the EXOR gate. If we can figure out a way to "time shift" one of them we will be able to see the correlation process at the output of the lowpass filter. Since our simple circuit uses a digital multiplier with an analog filter its neither pure analog or pure digital. The closest description is *Quasi Analog*.

### **4 Time Shifting or sliding one waveform with respect to another.**

Figure A3 shows two identical C/A code generators. If we use *slightly* different frequencies to clock these two generators they will appear to slide by each other in time. This is best observed by using a multi-channel scope. Generator #1 is on channel 1 and generator #2 is on channel 2. We trigger the scope on the repeat time of GEN#1 (the C/A Epoch Pulse). This test set up is shown in figure A3. Assuming the two clocks for the generators are not equal the code output from generator 2 will appear to slide right or left with respect to the code output of GEN#1 (Ch. 1). *The sign of the frequency difference between the two code clocks will determine the direction of movement, The rate of movement by the magnitude of the difference of the two clocks. If the two frequencies are exactly equal no movement will be seen but a fixed phase offset of the two C/A codes most likely would be present.*

The reader should understand that if we multiply these two waveforms together point by point and then take the average value we would only have a non zero average value when the two codes are within two bits (or chips) of alignment. At all other points the resulting waveform from the multiplication process has just as many +1 's as -1 's so the average is zero. When we have perfect alignment the average value would be +1.

## 5 Correlation Pulse and A Delay Based Discriminator

Figure A4 shows two identical EXOR based correlators and two identical C/A code generators. As before we are offsetting generator#2 code clock slightly so as to slide the C/A code w.r.t. Generator#1. Channel one of scope shows the output of correlator #1. As the two codes come within two bits of alignment (two chips) the output starts to rise. As they slide further a max value of +1 is achieved and then we decrease again to zero. Correlator #2 is slightly different in that a delay of one clock time(1 chip) has been introduced into C/A code from generator #1. The delay can be implemented a number of ways, delay lines, logic gates, clocking etc. This delay has the effect of moving the correlation peak from correlator#2 to right, or later in time, as shown on channel#2. Channel 3 of the scope shows the difference waveform obtained by subtracting (point by point) channel 2 from channel 1. This function can be done simply with an OP Amp. Here we assume the scope does this function.

The Channel 3 waveform is crucial to the GPS receiver's C/A code tracking function. It is a delay-based discriminator (or error voltage) and it forms the heart of the code-tracking loop. If we apply the error voltage properly to the Oscillator control point of Gen#2 we can force Gen#2 C/A code to stay aligned with the C/A code from Gen #1. That is we can lock Gen#2 to Gen#1. This error voltage would obtain code lock at ½ max correlation point. This can be remedied by adding another correlator with a delay of ½ chip. This correlator will now be at max value when use the error voltage channel #3 to lock generator#2 to Generator #1.

The C/A code repeats every 1023 bits(chips) so the waveforms of figure A4 will repeat in time. The repeat rate of the correlation/discriminator waveforms is:

$$\text{The Period of the Sliding Correlator Peaks} = 1023 / \Delta F$$

Where  $\Delta F$  is the frequency difference between code clock Gen#1 and code clock Gen#2. For the C/A code generator the code clock rate is 1.023Mhz. Typically  $\Delta F$  (in analog receivers) is in the 1 to 10hz range so it's a very small fraction of the code clock frequency. The width of the correlation peak is also determined by the clock frequency difference;

$$\text{Correlation Pulse Width} = [2 * (\text{DELAY} * \text{CODE CLOCK FREQ})] / \Delta F$$

Where DELAY is in seconds and Code Clock Frequency is in hertz. Typically a one-chip delay is used. A one-chip delay is equal to one code clock period. Therefore for this case;

$$\text{CODE CLOCK PERIOD} * \text{CODE CLOCK FREQ} = 1$$

$$\text{and Correlation Pulse Width} = 2/\Delta F$$

## 6 The Error Voltage or Discriminator Output

The error voltage shown on channel 3 of our scope is shown in more detail in figure A5. Instead of time on the x axis the unit of CHIPS is used. One chip is equivalent to 1 bit of C/A code. Therefore the C/A code has 1023 individual chips. From figure A5 we see that the error voltage is only linear over a 1 chip transition. Outside this it reverses sign and returns to zero volts. This lack of range means that a “hunting” technique must be used to get close to lock point and then allow lock to occur. Hence the sliding correlator. In practice the receiver slides its code in one direction at a steady state until correlation is detected, i.e. a fixed voltage at Gen#2 controls its oscillator. The receiver then allows the error voltage to control Gen#2 oscillator control thereby initiating C/A code lock.

The linear section of the error waveform can be used to correctly command the frequency of Gen#2 clock oscillator so as to maintain lock. If the error voltage is just *above* null point the frequency of Gen#2 clock is *increased*. If the error voltage is just *below* null point the frequency of Gen#2 clock is *decreased*. It may be that an inversion is present and in this case the above two statements are reversed in polarity of response to error voltage. Note that the discriminator error voltage provides both the *sign* of the error and its *magnitude*.

### Similarities to PLL

In many ways the C/A code correlation/discriminator/PN tracking system is equivalent to a carrier recovery system using a Phase Locked Loop. A PLL recovery of 1.023Mhz carrier using a 1khz comparison frequency is a good model to use for the GPS code clock recovery. In fact if the equations for second order PLL systems are properly re-scaled to reflect the delay discriminator they can be used predict the performance of the C/A code track function. The only real difference between the two systems is the phase detector, in the PLL it's usually a phase/freq. detector. In the PN code tracking systems it's a delay-based discriminator.

When one examines the quality the lock obtained on PN coded waveforms it is generally noisy or more jittery compared to a PLL approach. This is do partly to the quality of the phase detection scheme, the Delay Discriminator is a poor performer compared to the phase/ frequency detectors used in modern PLL systems. Modern phase/frequency detectors give error information on a cycle by cycle basis. A PN code based discriminator must have good replica code alignment over *many* bit edges in the entire period of the PN code to be effective.. Essentially in a PN code system we receive *phase error* information over *many cycles*( code clocks in this case) and not on a *cycle by cycle* basis as in the PLL system. Hence more jitter in PN coded systems for the same SNR.

### RC time Constant

Up till now we have not mentioned what values should be used for R and C of our simple correlators. These two components form a lowpass filter with a 3dB roll-off frequency of  $1/RC$ . The choice of this cutoff frequency, or time constant, will depend of the response desire from the system. If the filtered error voltage is used in a closed loop code tracking system then the time constant choice will be complex decision based on many considerations. Some of those are Loop Bandwidth, SNR of signal and Code Jitter in lock. Many receivers will use multiple RC time constants[ or digital equivalent] that are switched in depending on conditions in the receiver. Such a system uses the best loop bandwidth for the current condition in the receiver. For example scanning for code alignment performs better with a wider bandwidth. Once lock is established the bandwidth can be narrowed narrowed. Generally speaking wider bandwidths allow for faster lockup times but have more code jitter in the lock condition.

As a minimum RC time constant we need to integrate over the length of code used. For the C/A code that length is 1023 bits or approximately 1 msec period. Therefore the RC time constant should be a minimum of 1msec.

### Tau Dither Discriminator

Figure A6 shows a modified form of our delayed-based discriminator. Instead of two separate EXOR gates one gate is used. The Delay path is now switched in and out at a rapid rate by the dither clock. This switcthing of delay in and then out gives rise to the “dither” part of the name. After the EXOR is now bandpass filter, not a lowpass filter. This bandpass filter is tuned such that its center frequency is that of the dither clock rate. From here the output of the bandpass is amplitude detected and hard-limited. The hard-limited signal is EXOR'ed with the dither clock. The amplitude of bandpass filter output is the *magnitude*

of the error voltage. The EXOR output of the dither clock and the bandpass output is the *sign* of the error voltage. Both the amplitude detector and the sign EXOR detector are lowpass filtered before they are used. This removes any high frequency components that would be present from the operations of detection, limiting and EXORing. The scope shows the outputs of the magnitude and sign filters on channels 1 and 2. Channel 3 is the magnitude channel *multiplied* by the sign channel. The result Channel 3 shows the same discriminator curve shape we saw in the early late case.

Here is what is happening. As before we let the code from Gen#2 slide by code from Gen#1 by slightly offsetting Gen#2 code clock. As the code from Gen#2 is sliding by gen#1 code it is being dithered back and forth by the delay, typically about  $\frac{1}{2}$  chip. This dithering will produce an amplitude modulation waveform on top of a DC level at the they output correlator EXOR. This AM signal will be at the same frequency as the dither clock. The bandpass filter picks out this signal and passes it on to the detector (magnitude) and EXOR/lowpass filter (sign). The AM signal will undergo a 180 degree phase shift when the two codes pass the exact alignment point. This 180 degree phase shift contains the error voltage *sign* information and is recovered by EXORing the hard-limited bandpass output with dither clock and subsequent lowpass filtering. The output's are shown on the scope for the case where the delay is approximately  $\frac{1}{2}$  chip. The scope shows the output for the sign portion as zero outside the 2-chip correlation time window. This is an idealization. In a real circuit noise would cause this signal to behave erratically outside the two chip window, it would go between +1 and - 1 randomly.

The advantages of the tau-dither method are that it takes only one correlator to implement code tracking. Also amplitude imbalances that can be present in the two-corellator method are not a factor. In many PN code tracking loops it is the sign of the error that is used and the magnitude information is discarded. In this case the circuitry is simplified by eliminating the magnitude detector.

The delay based and tau-dither discriminators described above can both be implemented as carrier based systems. The advantages of the tau-dither method are more dramatic as carrier based correlators are significantly more complex than the base band versions.

## 10 Carrier Based Sliding Correlator

The tau-dither method just described is rarely used in the form outlined above. Rather it is used in carrier based system. Both the tau-dither and early late systems we have just covered are *baseband* correlators as no carrier (or IF) frequency was used. For the receiver described in this text a carrier based correlation method is used. In order to use the carrier based system requires that a sinewave be modulated with the C/A code so that its phase is switched between 0 and 180 degrees. Such a device is the BPSK modulator, See Appendix C. The BPSK modulator is the basic building block of the carrier based sliding correlator. BPSK Modulators are reciprocal devices in that they can both modulate and demodulated binary phase shifted carriers. It is the “undoing” of the phase modulation function of BPSK modulators we seek to employ here.

Figure A8 shows a sliding carrier based correlator. Once again we have two C/A code generators with GEN#2 clock rate slightly offset from clock of GEN#1. The major difference from before is that GEN#1 C/A code is modulating sinewave carrier. This output models the GPS transmitter code modulation of the carrier with no 50hz data present. This signal is now fed into another multiplier where GEN#2 code is applied. The output of this second multiplier is now bandpass filtered with a filter that has a center frequency at the carrier frequency used. When the replica C/A code is within 2 chips of the reference C/A code the output of the bandpass contains a sinewave. The amplitude of this sinewave grows then decays as the two codes slide in and out of correlation. This is indicated in Fig. 10 by the Amp/Freq/Time plot and the carrier in the diamond shaped envelope.

The principle of operation is very simple. Once the first multiplier modulates the carrier, the second multiplier will completely *remove* the carrier modulation *if* the C/A code of GEN#2 is perfectly lined up with the C/A code of GEN#1. In this condition the output of the second multiplier is the original sinewave(at the input of first multiplier)along with some artifacts from the 2<sup>nd</sup> multiplier. The bandpass filter serves roughly the same function as the lowpass filter did in the baseband correlators, as an integrator and a to filter out unwanted multiplier artifacts. For those familiar with Fourier transform recall that multiplication by a sinewave results in a frequency shift in the frequency domain. So the baseband signal spectrums are shifted up by the carrier frequency as well as the lowpass filter operation. A frequency shifted lowpass filter is just a bandpass filter.

When the C/A codes from GEN#1 and GEN#2 are not aligned (> two chip offset) the output of the bandpass filter would be the power spectrum a BPSK modulated carrier. But in a real world receiver the output of the bandpass filter is noise when the codes are not aligned. The output of the bandpass filter is best examined with a spectrum analyzer, not a scope as in the base-band case. If a spectrum analyzer is used to look a carrier based correlator the display will typically show a noise pedestal of bandwidth equal to the bandpass filter for the case where the codes are not aligned. When the codes are in near or perfect alignment a spike at the carrier frequency will be seen riding on top of the same noise pedestal. This is shown in figure A9.

## 11 A Carrier Based Tau- Dither Correlator/Desriminator

Figure A10 shows the Tau Dither Correlator using a carrier. The functions of the circuit are very similar to the carrier based system just discussed and the base band Tau Dither correlator covered earlier. As in the carrier based sliding correlator of Figure A8 the C/A code is modulated onto the sinewave carrier. The C/A Gen#1 and modulation onto the sinewave are integrated into the first block of figure A10. From here the signal enters the correlator. As above the multiplier can be a double balanced mixer or many other types as previously discussed. Gen#2 is offset in frequency by applying the DC control signal to its code clock oscillator. The post correlation signal is bandpassed with a filter whose center frequency is set to the carrier frequency. Typical bandwidths for this filter are in the 200 to 1khz range. The C/A code input to the correlator is dithered between a delayed version and a no delay version of the code. The dither clock frequency is typically in the 200 to 300 hz range for a analog receiver.

Once out of the correlation block the signal is AM detected (or RSSI function). This signal has an induced dither AM component . The following bandpass filter stage is tuned to the dither clock frequency so as to “pick off” this dither induced AM signal. In the figure the time waveform associated with the output of the correlator, is a sinewave in a diamond shaped envelope. This envelope is show with small AM signal riding on top of the envelope. This would only be valid for small delay values below 1/10 of a chip. Larger value would distort the diamond envelope from that shown in figure A11. Though small values of dither delay can be used a value between ¼ and ½ chip is more common in GPS receivers.

The output of the dither bandpass contains the information neede to create the discriminator funtion as before. The amplitude of the dither induced Am has the error Magnitude. The phase of the dither induced AM has the Sign of the error. As before these signals are lowpass filtered before use.

## 12 Processing Gain

A common term in spread spectrum system *Processing Gain*. Processing gain is realized at the output of bandpass filter of the sliding carrier based correlator when code alignment is achieved. Here’s why; When the receiver replica C/A code is *not* aligned with the transmitted C/A code the received GPS signal power at the output of the bandpass filter is spread over approximately 2mhz of bandwidth ( center lobe of C/A BPSK spectrum is approx 2 Mhz) . When the receiver C/A code *is* aligned with transmitted code the signal power at the bandpass output is now squished into approximately 100hz of bandwidth ( Center lobe of 50hz random data spectrum). A rule of thumb is to use the ratio of these two bandwidths as the processing gain. Processing gain is remarkable, it is as if the signal is amplified *without also amplifying the noise* at the same time!

$$\text{Processing Gain} = \text{Bandwidth of Uncorrelated Signal} / \text{Bandwidth Correlated Signal}$$

For a GPS receiver this works out to (in db)

$$\text{Processing Gain GPS Rec.(db) } = 10 \log [ 2\text{Mhz} / 100\text{Hz} ] \rightarrow +43\text{dB}$$

This number is an estimate of processing gain. A more accurate estimate is the ratio of the SNR before and after correlation. In addition various imperfections in the correlator may also degrade this gain. Regardless the processing gain is large in a GPS receiver and enables the negative SNR environment before correlation to be turned into a positive SNR condition after correlation assuming typical bandwidths before and after.

## 14 Recovery of signal with Negative SNR

The spreading of GPS signal power by the C/A code can lead to the condition where the signal is below the noise floor in one part of the receiver and above it in other sections after processing. When the signal is below the noise floor at a given point in the receiver it has a *negative* SNR.

For a user at the earth's surface the received power from the GPS signal is very low. The specified minimum power at the earth's surface is approximately  $-130\text{dBm}$ . This power is the unmodulated carrier power in one hertz of bandwidth. When the signal is modulated with the C/A code and 50Hz data this power is spread over a larger bandwidth. Once the signal power is spread over 2Mhz the crest of C/A code spectrum is well below the  $-130\text{dBm}$  unmodulated carrier power. Ignoring the 50Hz data modulation (data mod off) the carrier power is spread over approximately 2mhz of bandwidth as 1khz equal spaced tones. The power level of these tones in the main lobe is fairly flat at about  $-30\text{dB}$  down from the pure carrier level. This puts these lines at approximately  $-160\text{dbm}$ .

A typical post correlation bandwidth for an analog GPS receiver is 1khz. For a perfect receiver this would put the noise floor at approx.  $-143\text{dBm}$ . If we do not have correlation (C/A codes not aligned) the 1khz tones will be well below the noise floor at this point in the receiver resulting in a negative SNR condition. Once we have correlation the signal power is restored to nearly the unmodulated power level and for 1khz bandwidth we would see a SNR of about  $+13\text{db}$ . So just because a negative SNR condition is present does not mean the GPS signal is "gone". It is just hiding under the receiver noise floor waiting for the power of the correlation operation to "resurrect" the signal. The exact SNR after correlation will depend on where in the receiver one is talking about and the effective noise bandwidth at that point. In the data demodulator where the data is stripped off the carrier the noise bandwidth can be quite narrow and higher SNR's are achieved, typically  $20\text{db}$  to  $30\text{db}$ .

The phenomenon of retrieving a negative SNR signal is hard to swallow. It is the authors feeling that this difficulty can be overcome by understanding what noise is and how it interacts with *discrete* signals. Noise from a spectral point of view is a density. It is not discrete. Therefore you cannot talk about noise levels or powers quantitatively without specifying the bandwidth that is related to the noise being measured (ie the circuit bandwidth). Noise power goes up as bandwidth widens. A discrete signal is completely different. Discrete signals, such as pure sine waves, have a power (or level) that is *independent* of the bandwidth of the circuit. Theoretically if one measures the power of a pure sinewave through a bandpass filter whose center frequency equals the sinewave frequency the power measured *is independent* of filter bandwidth. As we just stated this NOT the case with noise.

As stated above the power spectrum of the GPS C/A spectrum is made up of discrete spectral lines spaced at the code repeat rate of 1khz for the case where we have only C/A code modulation (Data =1 or 0 forever). These lines trace out the  $[\sin(x)/x]^2$  spectrum. Therefore the spectrum of the C/A modulated signal is not a density but really a collection of discrete signal lines. What would happen if looked at the received signal (before correlation) with an ideal spectrum analyzer that could use extremely narrow bandwidths?

Figure A11 shows an ideal spectrum analyzer display of the GPS carrier @1575.42mhz both unmodulated and with C/A code modulation on it (No 50Hz Data Mod). The power levels shown reflect those at the earth's surface using a 0db-gain antenna. The unmodulated carrier is shown shaded. Note that once C/A code modulates the carrier, the carrier is suppressed and there would be no signal at that position (in an ideal modulator). Our ideal analyzer does not contribute to the received noise power and has some very narrow Resolution Bandwidths (RBW'S). For each RBW the theoretical noise floor of the instrument is shown. Note that at a 1khz RBW the C/A tones are well below the noise floor of the instrument. As the RBW is narrowed the noise floor drops by  $10\text{Log RBW}$  but the discrete C/A modulation tones remain constant in power/amplitude. At a RBW of 0.1hz we would be able to see the spectral lines of the C/A modulated carrier *above* the noise floor. So in theory it is possible to see the GPS signal before correlation.

It is hoped that this example makes it clear the signal is still there with negative SNR conditions. It is all related to the circuit bandwidth and its effect on noise power. The processing gain of correlation makes it possible to recover the signal and its data by compressing the received signal power into a smaller bandwidth (allowing a reduction in the circuit bandwidth) thereby creating a positive SNR condition.

FIGURE A1: AVERAGE VALUE OF TIME WAVEFORMS

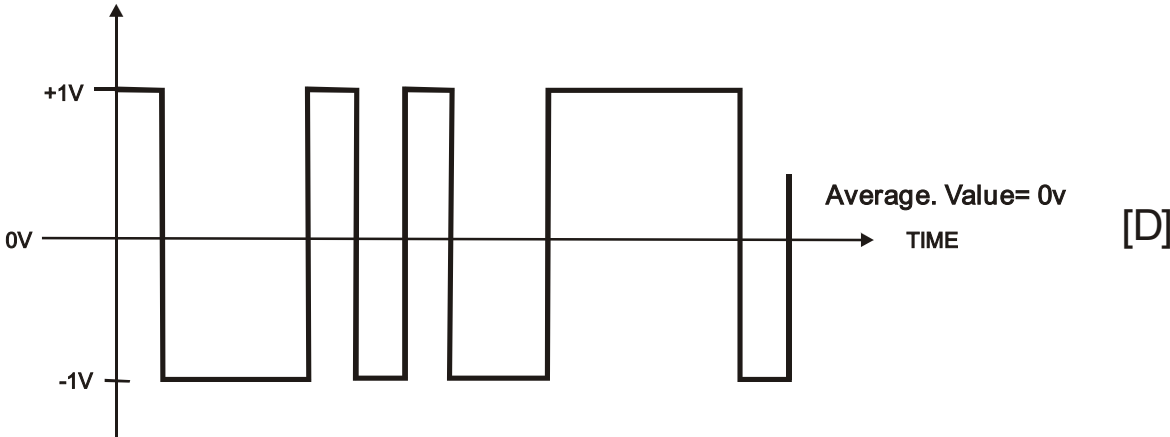
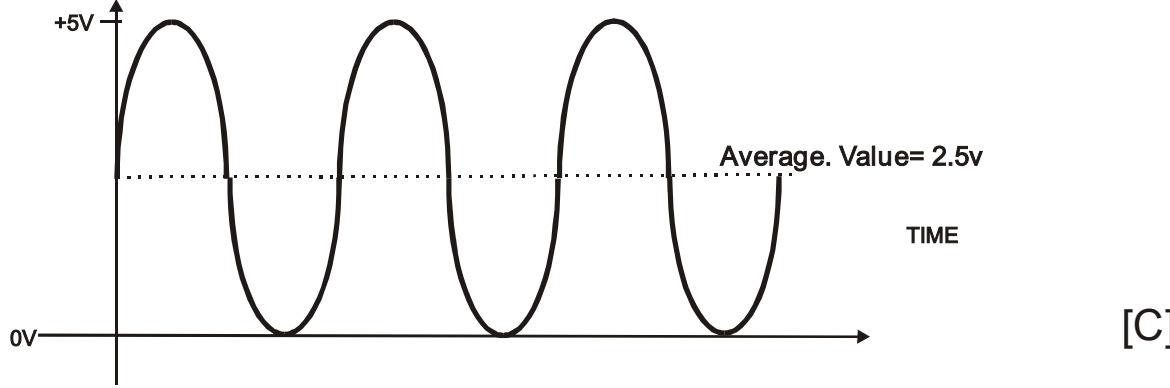
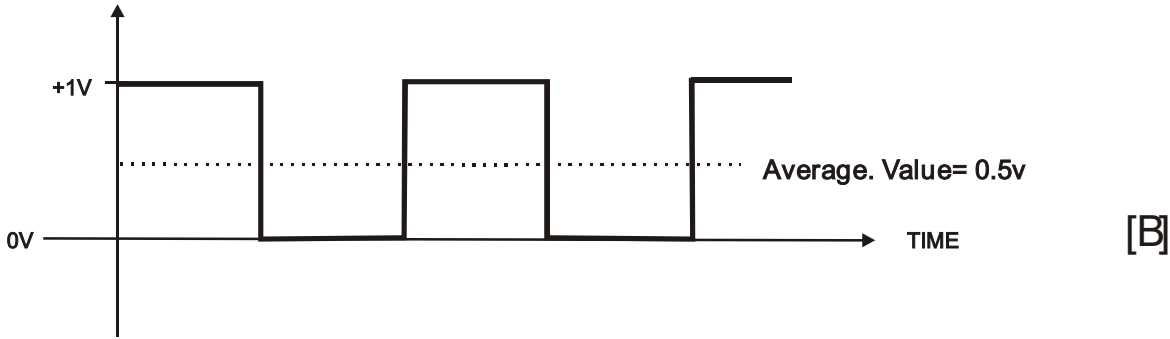
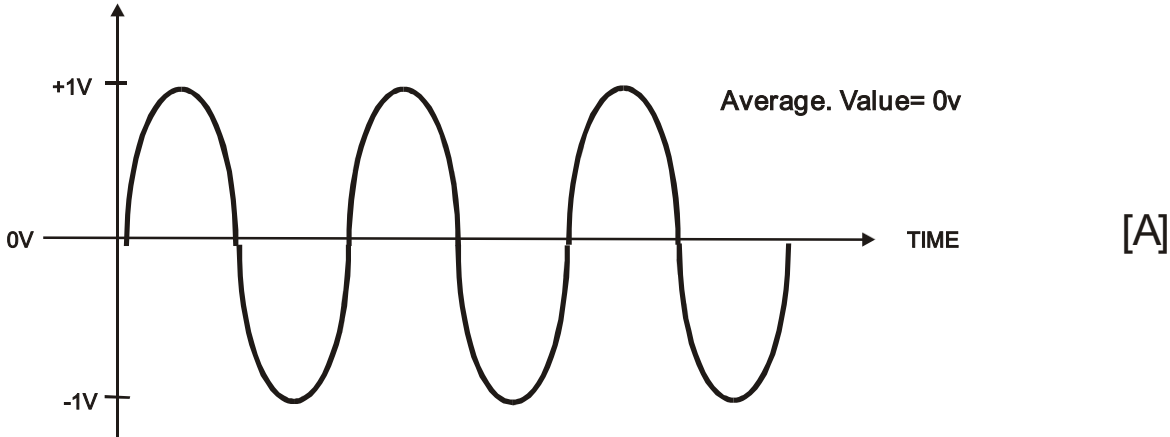
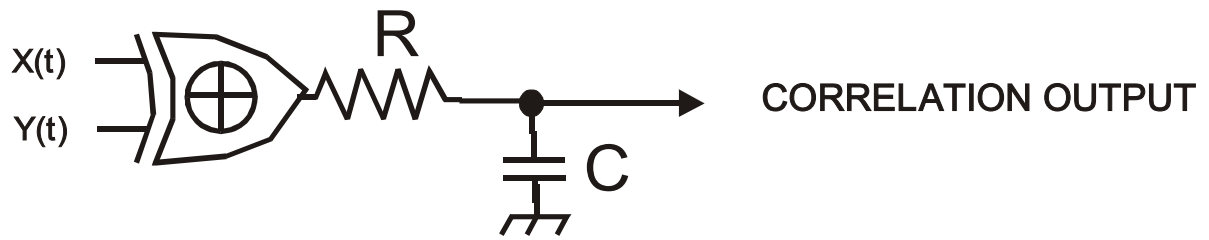


FIGURE A2: A QUASI-ANALOG CORRELATOR



BI- POLOAR  
EXOR

X	Y	Z
-1	-1	+1
+1	-1	-1
-1	+1	-1
+1	+1	+1

FIGURE A3:. CODE SLIDING BY DIFFERENCE IN CODE CLOCK FREQ

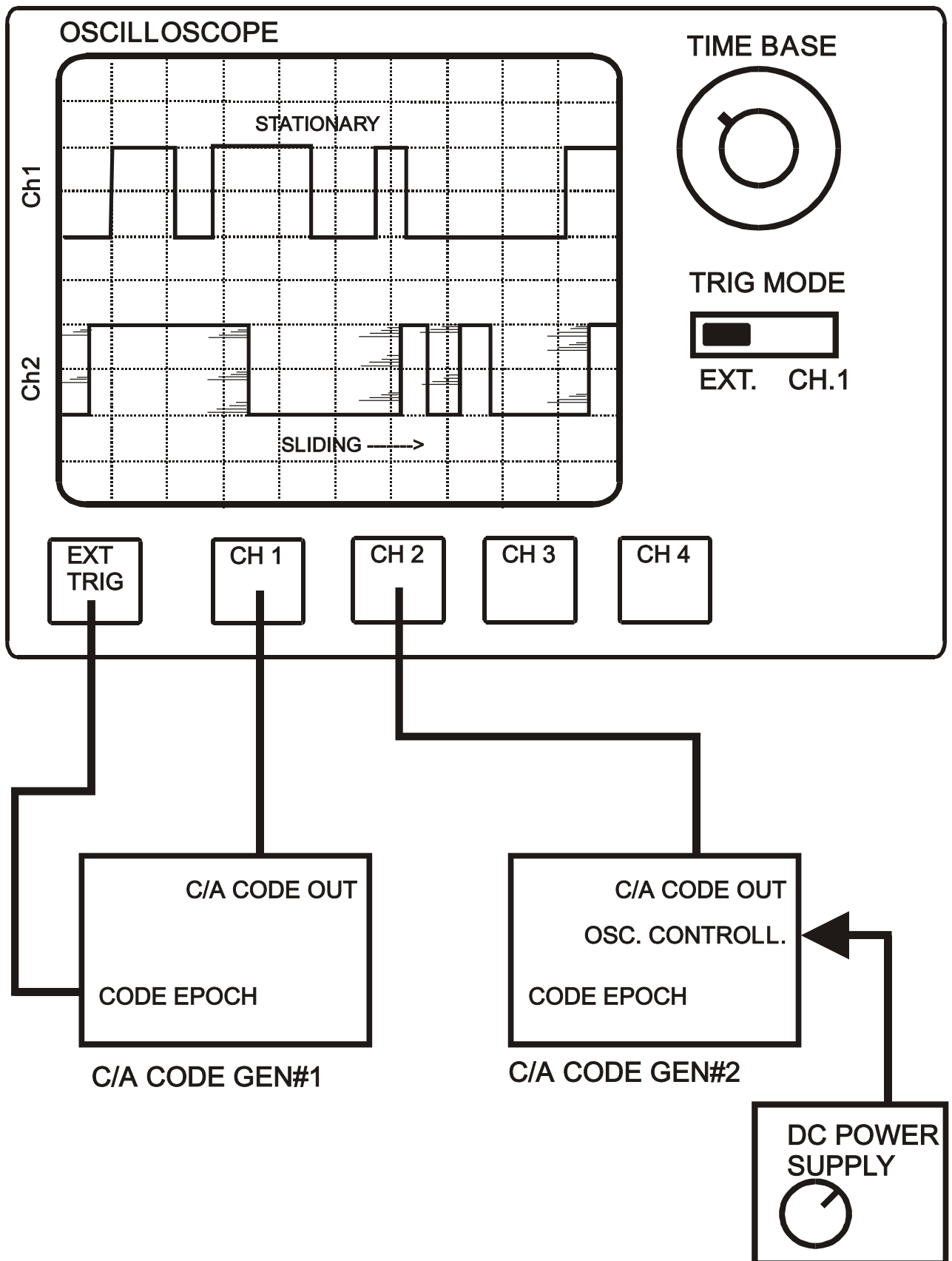




FIGURE A4. SLIDING CORRELATORS WITH 1 CHIP DELAY DISCRIMINATOR

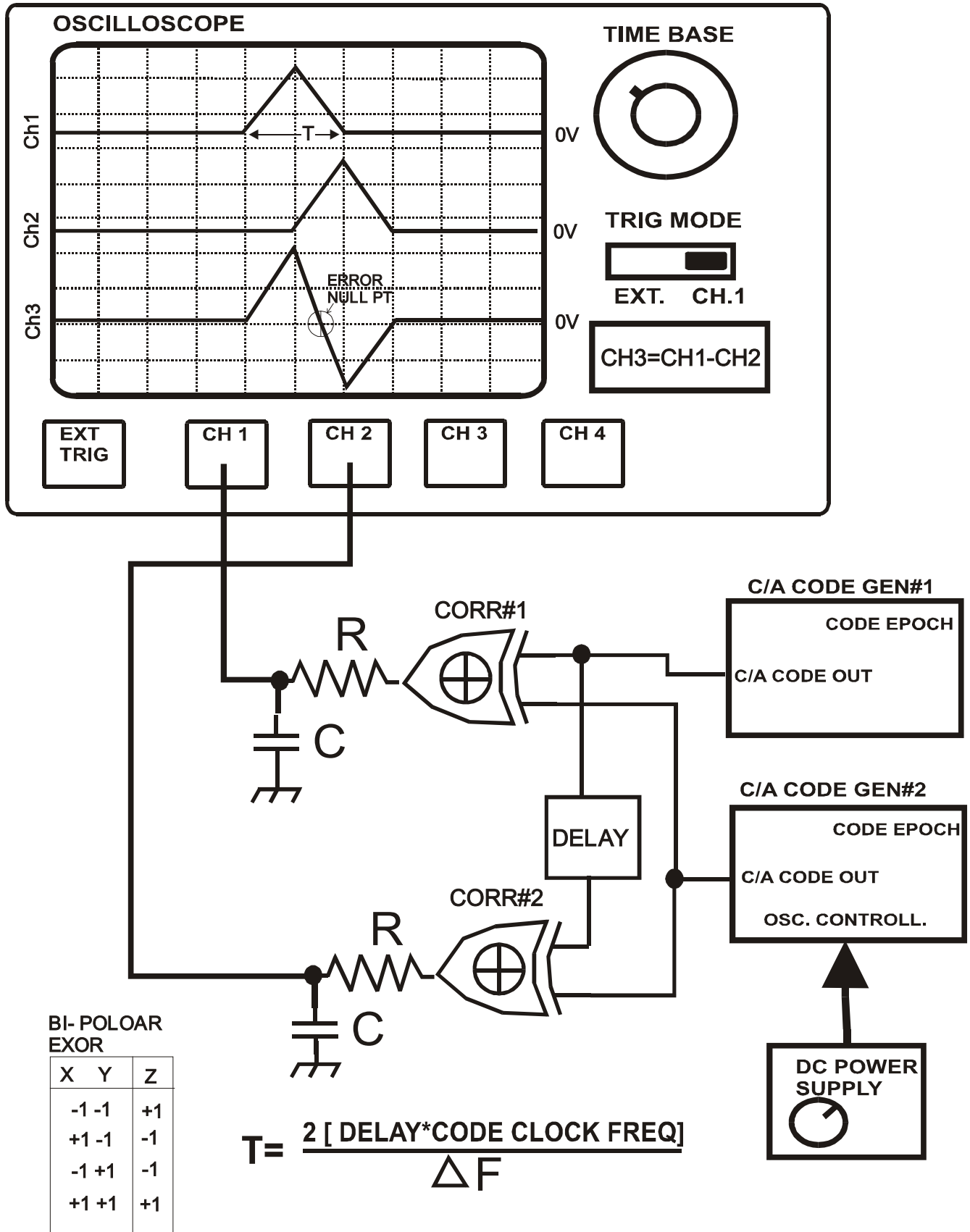
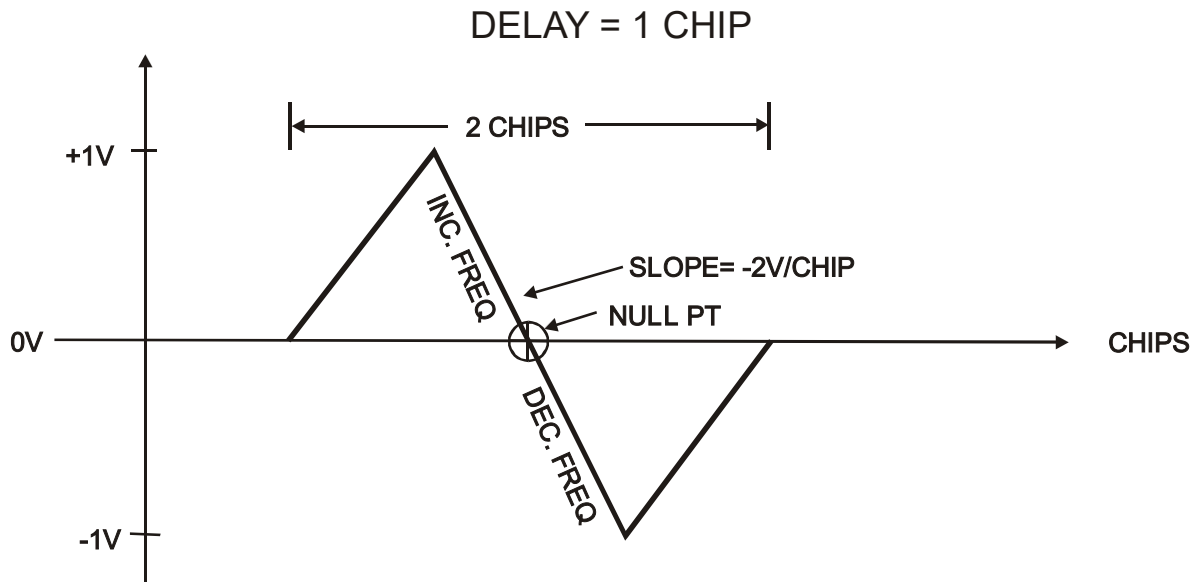




FIGURE A5: EARLY LATE DESCRIMNATOR CURVE



INCREASE/ DEREASE: COMMMAND POLARITY MAY INVERT  
DEPENDING VCO GAIN AND OTHER INVERSES OF ACTUAL  
CIRCUIT..

FIGURE A6: TAU DITHER CORRELATOR/DESCRIMINATOR FOR THE CASE WHERE DELAY IS  $\frac{1}{2}$  CHIP

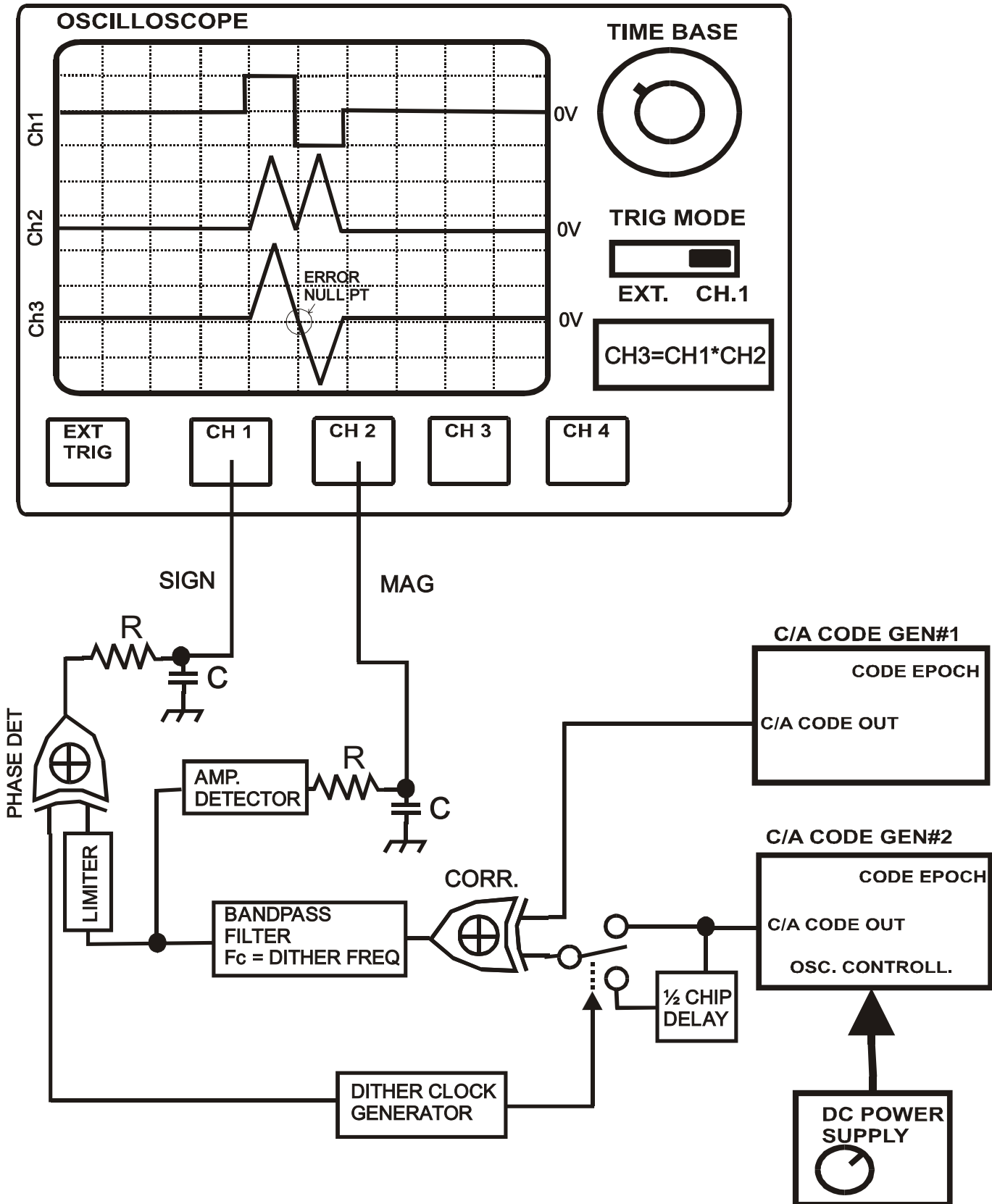
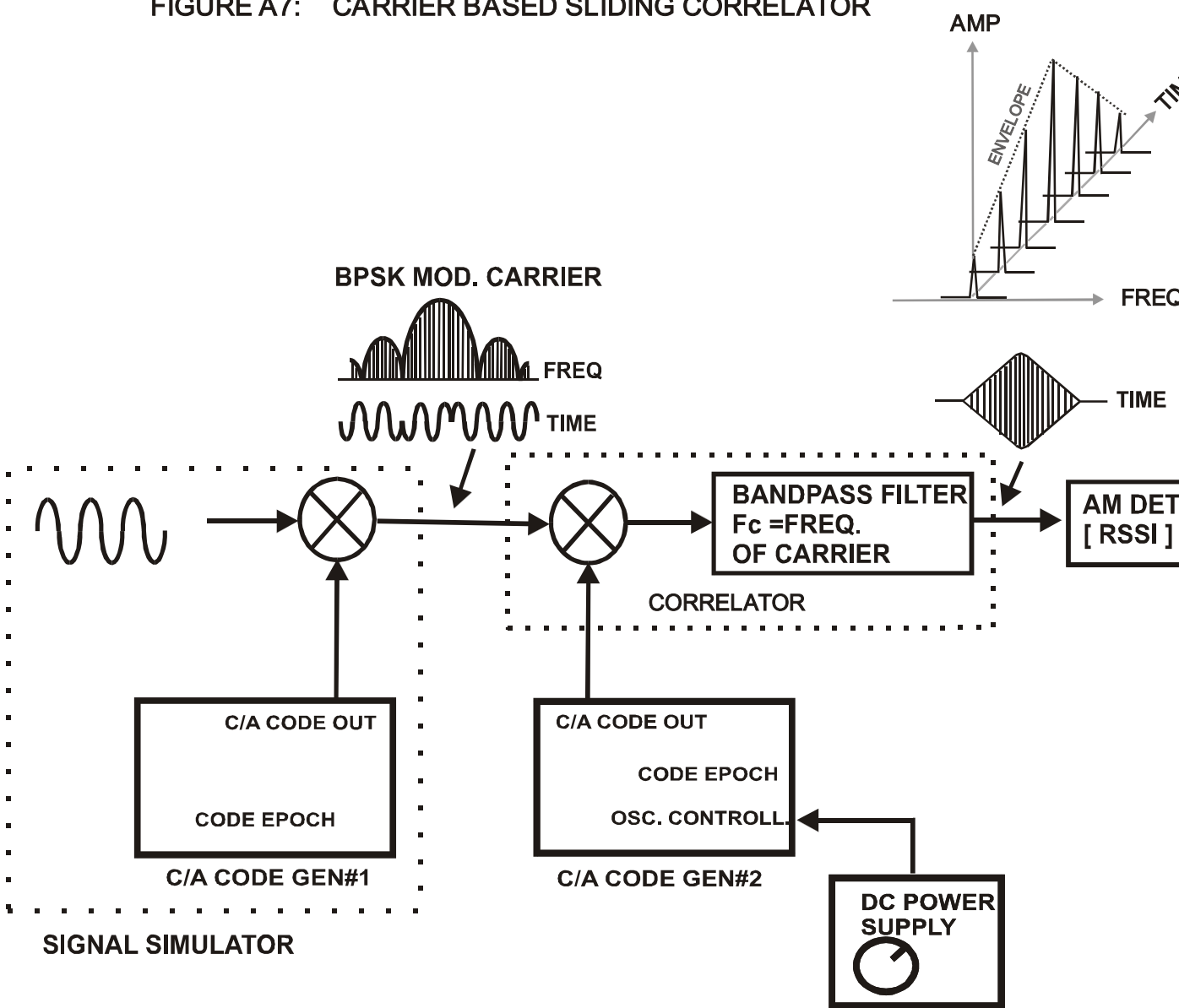
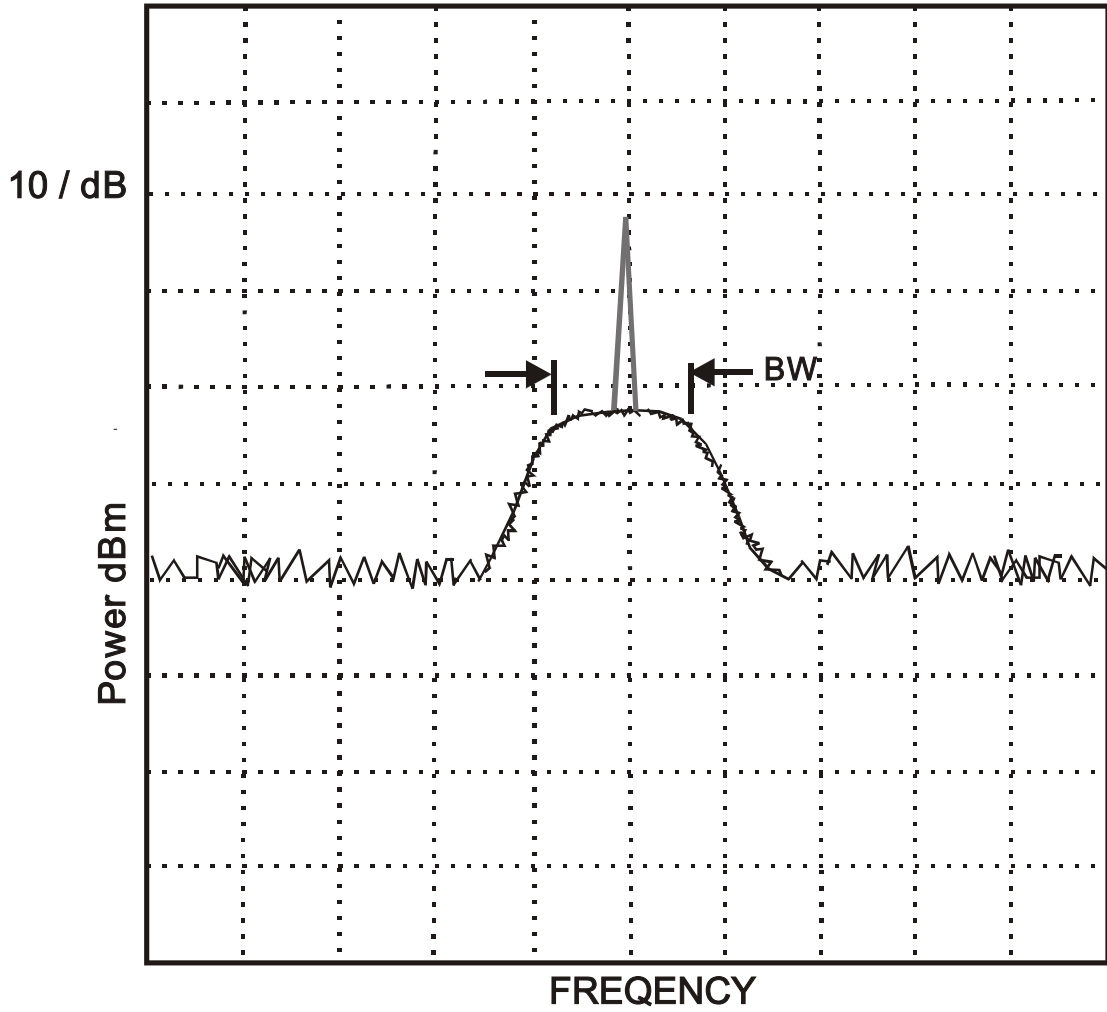




FIGURE A7: CARRIER BASED SLIDING CORRELATOR

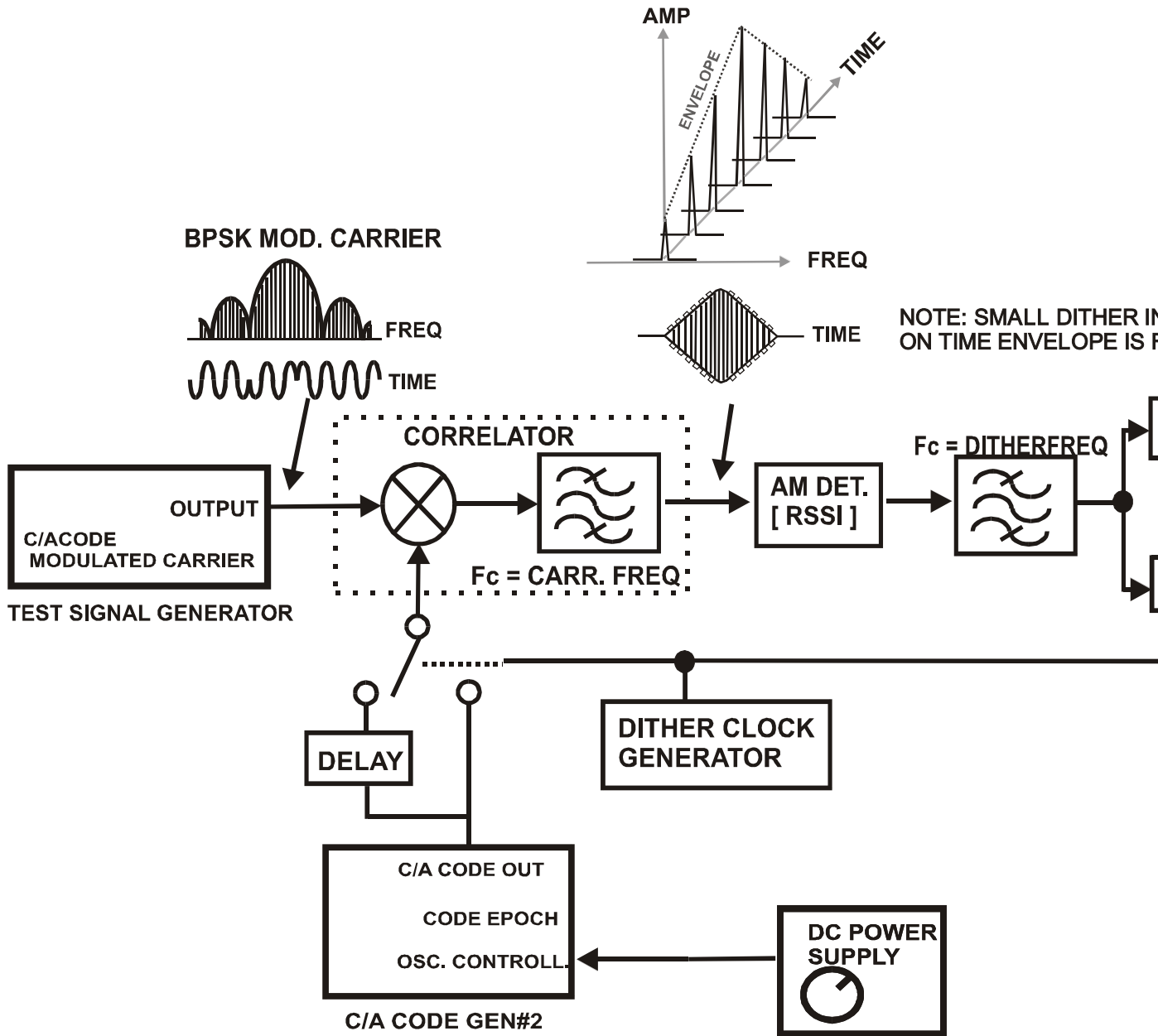


**FIG. A8: POST CORRELATION OUTPUT  
of BANDPASS FILTER**

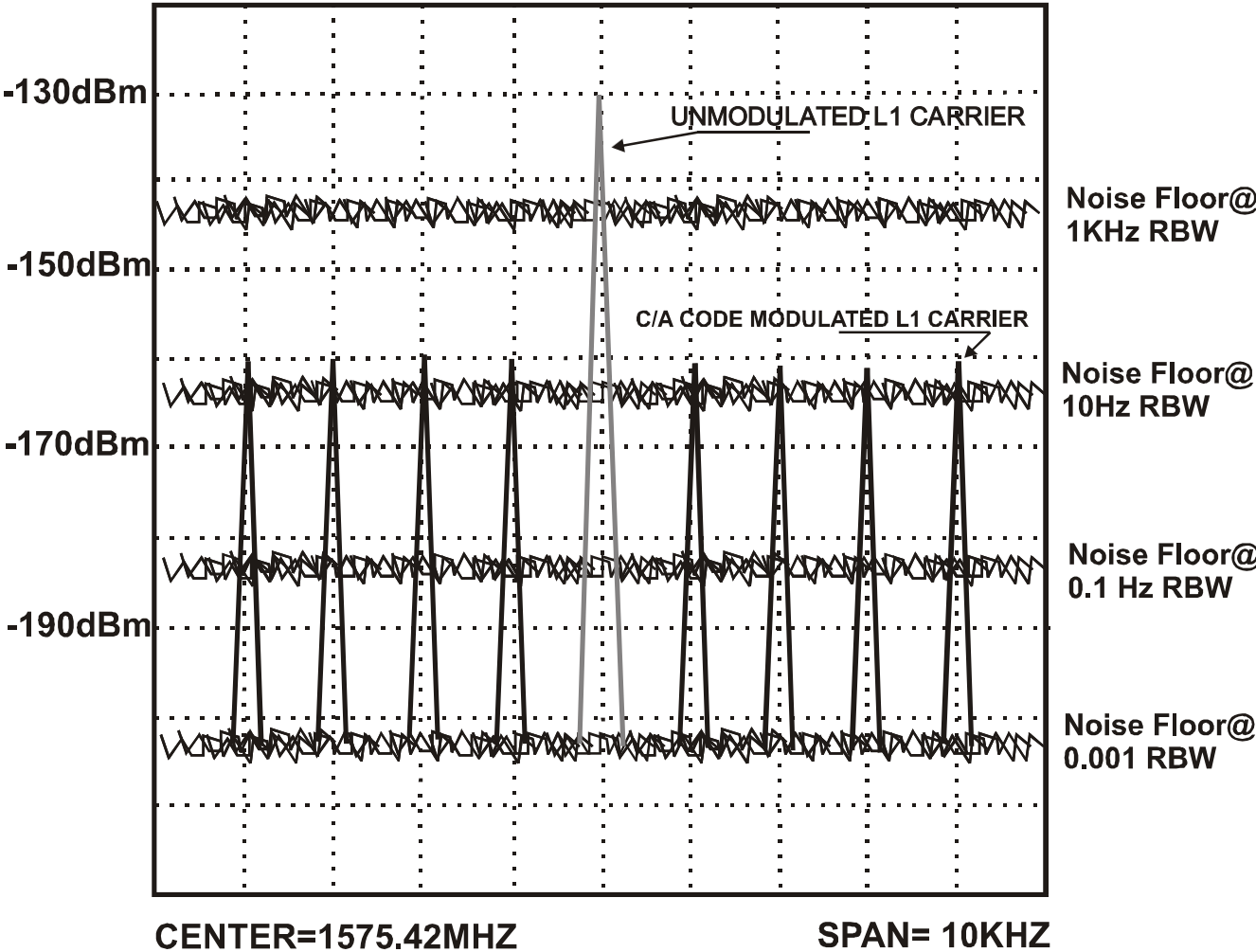


**BW = POST CORRELATOR BANDPASS FILTER**

FIGURE A9: SLIDING CORRELATOR & DISCRIMINATOR , TAU DITHER METHOD



**FIG. A10: GPS CARRIER WITH AND WITHOUT C/A CODE MODULATION @ EARTH'S SURFACE**



- NOTE:**
- 1) UNMODULATED CARRIER IS SHOWN SHADED
  - 2) 1KHZ LINES ARE C/A MOD. CARRIER, NO 50 HZ DATA PRESENT
  - 3) 1KHZ SPACED LINES ARE APPROX -30dBc WRT UNMOD CARRIER
  - 4) POWER SHOWN IS TYPICAL FOR RECEIVER AT EARTH'S SURFACE