

# Origin of 1/f PM and AM Noise in Bipolar Junction Transistor Amplifiers

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**Abstract**—In this paper we report the results of extensive research on phase modulation (PM) and amplitude modulation (AM) noise in linear bipolar junction transistor (BJT) amplifiers. BJT amplifiers exhibit 1/f PM and AM noise about a carrier signal that is much larger than the amplifiers thermal noise at those frequencies in the absence of the carrier signal. Our work shows that the 1/f PM noise of a BJT based amplifier is accompanied by 1/f AM noise which can be higher, lower, or nearly equal, depending on the circuit implementation. The 1/f AM and PM noise in BJTs is primarily the result of 1/f fluctuations in transistor current, transistor capacitance, circuit supply voltages, circuit impedances, and circuit configuration. We discuss the theory and present experimental data in reference to common emitter amplifiers, but the analysis can be applied to other configurations as well. This study provides the functional dependence of 1/f AM and PM noise on transistor parameters, circuit parameters, and signal frequency, thereby laying the groundwork for a comprehensive theory of 1/f AM and PM noise in BJT amplifiers. We show that in many cases the 1/f PM and AM noise can be reduced below the thermal noise of the amplifier.

## I. INTRODUCTION

THE FOCUS of this paper is to present a theory which explains the up-conversion of the baseband (near dc) noise of amplifier circuits to produce phase modulation (PM) and amplitude modulation (AM) noise about a coherent high frequency signal in linear amplifiers which use silicon bipolar junction transistors (BJTs) as the active gain element. This theory is based on a large number of experimental measurements and theoretical modeling of BJTs. The theory provides a starting point for designing amplifiers with low PM and AM noise and for estimating how the noise scales with operating point and signal frequency. Several other papers have studied the up-conversion of baseband noise to PM noise in microwave oscillators from a phenomenological point of view [1]–[4]. In this paper, we seek to provide a physical basis for the up-conversion process for both AM and PM noise in linear Si BJT amplifiers. We have chosen to make this study in amplifiers instead of oscillators because the results are

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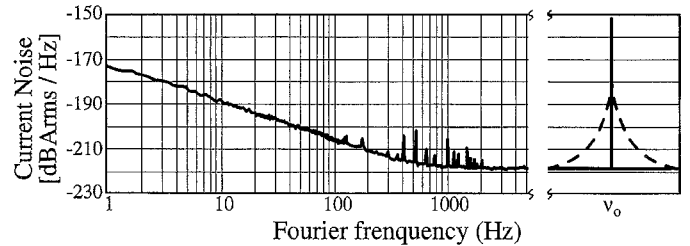


Fig. 1. Current noise of a CE amplifier as a function of frequency.

easier to understand and they can be extended to many different oscillator configurations.

Equation (1) shows the model of a perfect signal of frequency  $\nu_o$  and amplitude  $V_o$  with amplitude fluctuations characterized by  $\varepsilon(t)$  and phase fluctuations characterized by  $\phi(t)$ .

$$V(t) = [V_o + \varepsilon(t)] \cos(2\pi\nu_o t + \phi(t)). \quad (1)$$

PM noise is characterized by:

$$S_\phi(f) = [\Delta\phi(f)]^2 \frac{1}{BW}, \quad 0 < f < \infty, \quad (2)$$

where BW is the bandwidth of the measurement, and  $f$  is the Fourier frequency offset from  $\nu_o$ . The AM noise is characterized by

$$S_a(f) = \frac{[\Delta\varepsilon(f)]^2}{V_o^2} \frac{1}{BW}, \quad 0 < f < \infty, \quad (3)$$

where BW is the bandwidth of the measurement. These definitions show that one cannot define the added phase noise or amplitude noise of an amplifier except in the presence of a carrier signal. In much of the literature  $1/2S_a(f)$  and  $1/2S_\phi(f) = \mathcal{L}(f)$  are used to characterize AM and PM noise in devices and we will also use this notation for the remainder of the paper.

The solid line in Fig. 1 shows the output noise current per Hz bandwidth of a common emitter (CE) amplifier as a function of frequency. Near zero frequency the noise current typically varies as approximately 1/f where  $f$  is frequency. At higher frequencies (approximately 300 Hz for silicon BJTs), the noise power/Hz delivered to a matched load is independent of frequency and given by

$$P_N = kTFG, \quad (4)$$

where  $k$  is Boltzmann's constant,  $T$  is temperature in kelvin,  $F$  is the noise figure, and  $G$  is the power gain.

At 300 K this thermal noise is equal to  $-177$  dBm/Hz for  $F = G = 1$ . When a coherent signal is introduced, this thermal noise is divided equally between AM and PM noise and scales as  $1/P_o$ , where  $P_o$  is the output signal power:

$$1/2S_a(f) = \mathcal{L}(f) = kTFG/2P_o. \quad (5)$$

If the noise on the input signal is low enough, it is possible to show that the amplifier adds a small amount of noise power, in addition to the thermal noise given in (4), to the output signal. This excess noise, which is proportional to the input signal power, is illustrated by the dashed line in Fig. 1. When referenced to the center of the signal, this excess noise has the same  $1/f$  characteristics as that observed near dc and is therefore commonly referred to up-converted dc or baseband noise. *In contrast to the thermal noise, circuit parameters determine whether the  $1/f$  portion of the AM noise is higher, equal to, or lower than the  $1/f$  portion of the PM noise.*

In the following we present a theory and experimental data to explain the creation of excess  $1/f$  PM and AM noise in amplifiers which use silicon BJTs as the active gain element. We find that the  $1/f$  PM and AM noise is primarily the result of  $1/f$  fluctuations in transistor current, transistor capacitance, circuit supply voltages, and circuit impedances. This theory shows the first order dependence of  $1/f$  PM and AM noise in linear amplifiers on circuit parameters, transistor parameters, and signal frequency and therefore can be used to select transistors and choose the circuit parameters that minimize added  $1/f$  noise in amplifiers. Although the theory is discussed relative to a CE configuration, it can also be used to analyze other configurations. In many cases the  $1/f$  noise can be reduced to such an extent that it is much lower than the thermal noise for frequency offsets from the signal larger than a few hertz [5].

## II. EARLY WORK ON $1/f$ NOISE IN AMPLIFIERS AND FREQUENCY MULTIPLIERS

Halford *et al.* [6] and Andressen and Nesheim [7] were the first to show that the  $1/f$  PM noise of CE amplifiers and frequency multipliers can be reduced by adding an unbypassed resistance  $R_E$  in the emitter leg of the transistor [6], [7]. See Fig. 2. The mechanisms by which the up-conversion occurred and the reason that only feedback to stabilize the rf gain provided significant reduction of the flicker PM noise were not understood. At the time of this early work, the excess AM noise was ignored by virtually everyone except Healy [8]. Because no theory existed to explain the reduction of the PM noise with unbypassed emitter resistor or the effect on the AM noise, it was not possible to design amplifiers with low  $1/f$  noise except by trial and error.

Our work in this area started several years ago with the publication of [9], [10]. This work showed that the AM

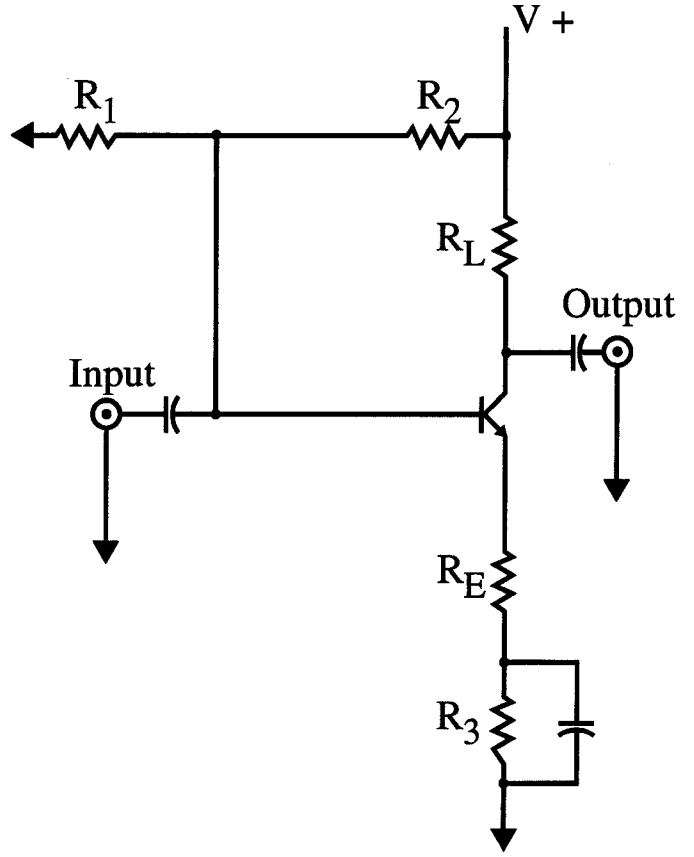


Fig. 2. CE amplifier with unbypassed emitter resistor.

and PM noise in amplifiers is often very similar and raised the possibility that the noise might be due to the same mechanisms.

## III. $1/f$ NOISE IN LINEAR AMPLIFIERS

We first derive the complete general expression for added AM and PM noise in an amplifier. The first order approximations to these expressions are then used to derive the added AM and PM noise for a CE amplifier with a single dominant pole. We then show in a simplified example how fluctuations in the emitter current cause fluctuations in the magnitude of the gain, thereby inducing AM noise. We then include the current and voltage dependence of various transistor and circuit elements to develop a more complete theory of both AM and PM noise in CE amplifiers.

## IV. A GENERAL EXPRESSION OF ADDED AM AND PM NOISE IN AMPLIFIERS

The general transfer function for an amplifier,  $G_V(s)$ , can be written as:

$$G_V(s) = G_o F_L(s) F_H(s), \quad (6)$$

where  $s = j\omega$  ( $\omega = 2\pi\nu_o$  = angular frequency),  $G_o$  is the mid-band voltage gain,  $F_L$  is the low frequency dependence of the transfer function, and  $F_H$  describes the high frequency dependence of the voltage gain on frequency. For the purposes of the present discussion, we can concentrate on the behavior of  $F_H$ , as we are assuming that the amplifier is operating at a frequency well above its low frequency cut-off.  $F_H$  can be written as:

$$F_H(s) = \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2}) \dots (1 + s/\omega_{znh})}{(1 + s/\omega_{p1})(1 + s/\omega_{p2}) \dots (1 + s/\omega_{pnh})}, \quad (7)$$

where  $\omega_{p1}, \omega_{p2}, \dots, \omega_{pnh}$  are positive numbers representing the angular frequencies of the  $n$  real poles and  $\omega_{z1}, \omega_{z2}, \dots, \omega_{znh}$  are real numbers representing the frequencies of the  $n$  zeros. (Note that these can be positive, negative, or infinite.) In many real-world cases the zeros are at such high frequencies that they are of little significance in determining the upper cutoff frequency of  $G_V(s)$ . In this case the function  $F_H(s)$  can be approximated by:

$$F_H(s) = \frac{1}{(1 + s/\omega_{p1})(1 + s/\omega_{p2}) \dots (1 + s/\omega_{pnh})}. \quad (8)$$

An alternative representation which will prove useful in the following discussion is to analyze  $G_V(s)$  in terms of the gain and phase. That representation leads to  $G_V(s)$  being described by:

$$G_V(s) = G_o |F_H(s)| e^{j\vartheta}, \quad \vartheta = \tan^{-1} \frac{\text{Im}(F_H(s))}{\text{Re}(F_H(s))}. \quad (9)$$

The two representations are equivalent.

Finally in the case which we will analyze in detail, the CE amplifier, one pole is dominant, at which point we can further approximate  $F_H(s)$  as:

$$F_H(s) = \frac{1}{1 + s/\omega_{p1}}. \quad (10)$$

(9) can now be written as:

$$G_V(s) = G_o \left( \frac{1}{1 + \delta^2} \right)^{1/2} e^{j\vartheta}, \quad \vartheta = \tan^{-1} \delta, \delta = \left( \frac{\omega}{\omega_{p1}} \right). \quad (11)$$

If we consider the input signal to be perfect, that is:

$$V_{in} = V_o \cos \omega t, \quad (12)$$

then the output signal is given by:

$$\begin{aligned} V_{out} &= V_{in} G_V = V_o G_o \left( \frac{1}{1 + \delta^2} \right)^{1/2} e^{j\vartheta} \cos(\omega t) \\ &= V_o G_o \left( \frac{1}{1 + \delta^2} \right)^{1/2} \cos(\omega t + \vartheta). \end{aligned} \quad (13)$$

The AM noise added by an amplifier can be written as:

$$\frac{1}{2} S_a(f) \cong \frac{1}{2} \left( \frac{\Delta G_V}{G_V} \right)^2 \frac{1}{BW} + \frac{kTFG}{2P_0}, \quad (14)$$

and the PM noise added by an amplifier can be written as:

$$\frac{1}{2} S_\phi(f) \cong \frac{1}{2} \Delta \vartheta^2 \frac{1}{BW} + \frac{kTFG}{2P_0}. \quad (15)$$

Equations (14) and (15) also include the contribution of the thermal noise. These expressions are completely general and can be used on any two port element such as an amplifier, attenuator, or level shifter, as long as the output frequency is the same as the input by expressing  $G_V$  as a function of the transistor and circuit parameters at the operating carrier frequency. Equations (14) and (15) can also be extended to cover two port elements where the frequency is changed by multiplying by the square of the ratio of output to input frequency.

## V. SINGLE POLE APPROXIMATION FOR GAIN OF COMMON-EMITTER AMPLIFIER

In this section we use the above results to develop a physical model which explains the up-conversion of base-band noise to create excess AM and PM noise in CE amplifiers. The equation describing gain in a practical CE amplifier is very complicated; therefore, several approximations will be used to simplify the equations so that the underlying principles are more easily seen.

In the dominant pole case at frequencies  $\omega$  much smaller than the 3 dB bandwidth ( $\omega_{p1}$ ), that is  $\delta = \left( \frac{\omega}{\omega_{p1}} \right) < 1$ , (11) can be further approximated and simplified to

$$\begin{aligned} G_V &= G_o \left( \frac{1}{1 + \delta^2} \right)^{1/2} e^{j\vartheta} \\ &= G_o \left( 1 - \frac{\delta^2}{2} + \frac{3\delta^4}{8} \dots \right) e^{j(\delta + \delta^3 \dots)} \\ &= G_o \left( 1 - \frac{\delta^2}{2} \right) e^{j\delta}. \end{aligned} \quad (16)$$

This result can be used to simplify (13) to

$$V_{out} = V_{in} G_V = V_o G_o \left( 1 - \frac{\delta^2}{2} \right) \cos(\omega t + \delta). \quad (17)$$

Equation (14) can be expanded to yield

$$\frac{1}{2} S_a(f) \cong \frac{1}{2} \left( \frac{\Delta G_o}{G_o} \right)^2 \frac{1}{BW} + \frac{1}{2} \delta^2 \Delta \delta^2 \frac{1}{BW} + \frac{kTFG}{2P_0}. \quad (18)$$

Equation (18) can also be expressed as

$$\begin{aligned} \frac{1}{2} S_a(f) &\cong \frac{1}{2} \left( \frac{\Delta G_o}{G_o} \right)^2 \frac{1}{BW} + \frac{1}{2} \delta^2 \left( S_\phi(f) - \frac{kTFG}{2P_0} \right) \\ &\quad + \frac{kTFG}{2P_0}, \end{aligned} \quad (19)$$

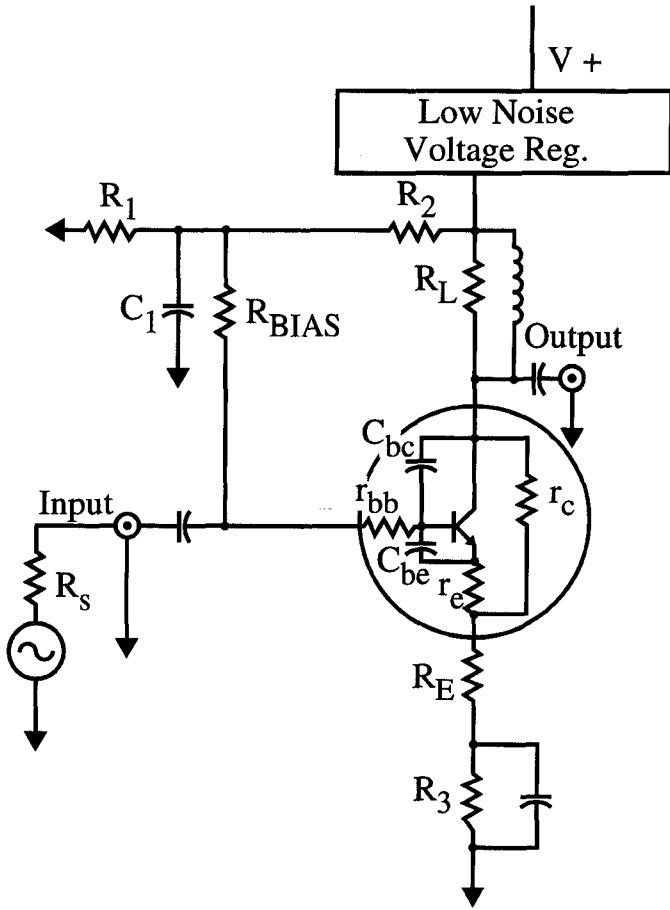


Fig. 3. CE amplifier showing transistor model.

where

$$\frac{1}{2}S_{\phi}(f) \cong \frac{1}{2}\Delta\delta^2 \frac{1}{BW} + \frac{kTFG}{2P_0}. \quad (20)$$

Fig. 3 shows the schematic of a simple CE amplifier. In the approximation that the load impedance can be represented by  $R_L$  and that  $\omega$  is much smaller than  $\omega_{p1}$  (that is, the phase shift  $\delta$  is small), we can write the gain as:

$$G_V = G_o \left( \frac{1}{1 + \delta^2} \right)^{1/2} e^{-j\vartheta} = G_o \left( 1 - \frac{\delta^2}{2} \right) e^{-j\delta}, \quad (21)$$

$$G_o = -\frac{R_L}{r_e + R_E + r_g/\beta}, \quad r_g = \left( r_{bb} + \frac{R_S R_{BIAS}}{R_S + R_{BIAS}} \right), \quad (22)$$

$$\delta \cong \omega C_{bc} |G_o| (r_e + R_E + r_g) + \omega C_{be} |G_o| (r_g + R_E)/R_L + \dots \quad (23)$$

where the  $R$ 's are defined in Fig. 3,  $r_g$  is the total effective input impedance, and  $\delta$  is the phase shift due to  $C_{bc}$  and  $C_{be}$ . A further simplification can be made for CE amplifiers with large gain or  $C_{be} \frac{r_e}{R_L} \ll C_{bc}$ . In this case, the gain

becomes

$$G_o \cong -\frac{R_L}{r_e + R_E + r_g/\beta}, \quad (24)$$

$$r_g = \left( r_{bb} + \frac{R_S R_{BIAS}}{R_S + R_{BIAS}} \right), \quad (25)$$

$$\delta \cong \omega C_{bc} |G_o| (r_e + R_E + r_g). \quad (26)$$

When the emitter current of a silicon BJT is noise modulated as:

$$I_E = I_{dc} + \Delta I_E \cos(\Omega t + \theta), \quad (27)$$

the intrinsic emitter resistance  $r_e$  is also modulated. For  $\Omega \ll \nu_o$  and  $\gamma = \frac{\Delta I_E}{I_E} \ll 1$ , the average value of  $r_e$  is given by:

$$r'_e \cong \frac{K}{I_{dc}[1 + \gamma \cos(\Omega t + \theta)]} \cong r_e [1 - \gamma \cos(\Omega t + \theta)], \quad (28)$$

with  $K = 26\Omega/\text{mA}$  emitter current. We can now express the gain as:

$$G_o \cong \frac{-R_L}{r_e + R_E + r_g/\beta} \left[ 1 + \frac{\gamma r_e \cos(\Omega t + \theta)}{r_e + R_E + r_g/\beta} \right]. \quad (29)$$

Averaging over the phase of the modulation yields

$$\overline{G_o^2} \cong \left( \frac{R_L}{r_e + R_E + r_g/\beta} \right)^2 \left[ 1 + \frac{\gamma^2}{2} \left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \right]. \quad (30)$$

The single sideband AM noise spectrum added by the amplifier due to current modulation follows the spectrum of the current modulation and is given by:

$$1/2 S_a(f) \cong 1/4 \left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \gamma^2(f). \quad (31)$$

Considering the possible variation of the other parameters affecting the gain and phase of this simplified CE amplifier, we obtain a more complete expression for the AM noise given by:

$$\begin{aligned} \frac{1}{2} S_a(f) \cong & \frac{1}{4} \left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \gamma^2 \\ & + \frac{1}{4} \left( \frac{r_g/\beta}{r_e + R_E + r_g/\beta} \right)^2 \left( \frac{\Delta r_g}{r_g} \right)^2 \\ & + \frac{1}{4} \left( \frac{R_E}{r_e + R_E + r_g/\beta} \right)^2 \left( \frac{\Delta R_E}{R_E} \right)^2 \\ & + \frac{1}{4} \left( \frac{\Delta R_L}{R_L} \right)^2 + \frac{\delta^2}{4} \left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \\ & \times (\omega C_{bc} G_o(r_g))^2 \gamma^2 + \frac{kTFG}{2P_0} \\ & + \frac{\delta^2}{4} \left[ (\omega G_o(r_e + R_E + r_g))^2 \Delta C_{bc}^2 \right. \\ & \left. + (\omega C_{bc} G_o)^2 (\Delta R_E^2 + \Delta r_g^2) \right]. \end{aligned} \quad (32)$$

Equation (32) can be considerably simplified if we make use of the following approximations,

$$r_g > (r_e + R_E), \Delta r_e = \gamma r_e. \quad (33)$$

$$\begin{aligned} \frac{1}{2}S_a(f) \cong & + \frac{G_o^2}{4} \left[ \left( \frac{\gamma r_e}{R_L} \right)^2 + \left( \frac{\Delta r_g}{\beta R_L} \right)^2 + \left( \frac{\Delta R_E}{R_L} \right)^2 \right] \\ & + \frac{1}{4} \left( \frac{\Delta R_L}{R_L} \right)^2 + \frac{kTFG}{2P_0} + \frac{(\omega G_o)^4 (C_{bc} r_g)^2}{4} \\ & \times ((r_g \Delta C_{bc})^2 + C_{bc}^2 (\Delta R_E^2 + \Delta r_g^2)) \end{aligned} \quad (34)$$

The first term in (34) scales as  $\left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2$ , while the contribution of the terms associated with  $\Delta C_{bc}$  scale as  $\left( \frac{r_g}{r_e + R_E + r_g/\beta} \right)^4$ . Both sets of terms are reduced by increasing  $R_E$ , although the AM noise originating from capacitance modulation is reduced much faster than that originating from current modulation. The first term is also decreased by increasing the dc emitter current  $I_E$ . The limit to which  $I_E$  can be increased depends on the maximum ratings of the transistor and its environment.

The PM noise added by the CE amplifier in the same approximation used above for the AM noise yields

$$\begin{aligned} 1/2S_\phi(f) = \mathcal{L}(f) = & \frac{1}{4} (\omega G_o (r_e + R_E + r_g))^2 \Delta C_{bc}^2 \\ & + \frac{1}{4} (\omega C_{bc} G_o)^2 (\Delta R_E^2 + \Delta r_g^2) \\ & + \frac{kTFG}{2P_0} + \frac{1}{4} \left( \frac{r_e}{r_e + R_E + r_g/\beta} \right)^2 \\ & \times (\omega C_{bc} G_o (r_g))^2 \gamma^2. \end{aligned} \quad (35)$$

For  $\Delta r_e = \gamma r_e, r_g > (r_e + R_E)$ , (35) can be reduced to:

$$\begin{aligned} \mathcal{L}(f) = & \frac{1}{4} (\omega G_o)^2 \left[ (r_g \Delta C_{bc})^2 + (C_{bc})^2 \left( \frac{\gamma r_g r_e G_o}{R_L} \right)^2 \right] \\ & + \frac{1}{4} (\omega G_o)^2 (C_{bc})^2 (\Delta R_E^2 + \Delta r_g^2) + \frac{kTFG}{2P_0}. \end{aligned} \quad (36)$$

The PM noise due to capacitance modulation is reduced as  $\left( \frac{R_L r_g}{r_e + R_E + r_g/\beta} \right)^2$ , while the contribution of the terms associated with current modulation scale as:  $(r_e r_g R_L)^2 \left( \frac{1}{r_e + R_E + r_g/\beta} \right)^4$ . Both sets of terms are reduced by increasing  $R_E$  and reducing  $r_g$ , although the PM noise originating from current modulation is reduced much faster with  $R_E$  than those originating from capacitance modulation. Increasing the dc emitter current  $I_E$  primarily affects the term originating from current modulation.

Current modulation originates from the intrinsic baseband modulation generated within the transistor and by noise in the power supply. The modulation of  $C_{bc}$  is induced by baseband modulation of the base collector voltage. This baseband voltage modulation can originate from the current modulation and finite dc gain or from the power supply.

## VI. MEASUREMENT OF AM AND PM NOISE INDUCED BY CURRENT MODULATION IN A CE AMPLIFIER

Current noise was injected in the emitter of the transistor shown in Fig. 3 to test the functional form of (31). Current noise of order  $22 \times 10^{-12} A_{rms}^2/\text{Hz}$  was used, which was large enough that the induced AM and PM noise was usually above the noise floor of our measurement system. Table I shows the measured AM noise sensitivity to  $\gamma(\Delta I_E/I_E)$  and the predictions calculated using (31) for a 2N2222A and a microwave transistor (with much lower  $C_{bc}$ ) for  $R_E = 0 \Omega$  and  $10 \Omega$ . The characteristics of the transistors are given in Table II. Column A in Table I shows the measured AM noise sensitivity to  $\gamma$  as a function of Fourier frequency offset from a 5 MHz carrier for the two transistors with  $R_E = 0$ , while column B shows the calculated value for  $r_g/\beta \cong 1 \Omega$  and  $r_e \cong 1.5 \Omega$ . Column C shows the measured AM noise sensitivity with  $R_E = 10 \Omega$  and Column D shows the calculated values. The calculated and measured improvements in the AM noise sensitivity to  $\gamma$  due to increasing  $R_E$  from 0 to  $10 \Omega$  agree very well for both transistors.

Column E shows the measured PM noise sensitivity to  $\gamma$  for  $R_E = 0 \Omega$  while column G shows the PM noise sensitivity to  $\gamma$  for  $R_E = 10 \Omega$ . The calculated values using the last term in (35) are shown in columns F and H. For the 2N2222A, the calculated values are in agreement with the measured values. Equation (35) predicts that the PM noise (due to current noise) at  $R_E = 10 \Omega$  is reduced relative to the PM noise at  $R_E = 0 \Omega$  by the square of the reductions observed for the AM noise. The measured reduction of PM noise follows this relatively closely for the 2N2222A. The measured PM noise sensitivity to  $\gamma$  for the microwave transistor is shown in column E. This PM sensitivity should be reduced relative to that of the 2N2222A by the ratio of  $(C_{bc1}/C_{bc2})^2$ . For these two transistors that ratio is approximately 22 dB. The measured reduction is about 5 dB higher. Measurements at  $R_E = 10 \Omega$  for the microwave transistor were limited by the noise floor of the measurement system.

Table I along with (34) and (35) can be used to predict the AM and PM levels due to current fluctuations in a CE amplifier. For a given current noise ( $\Delta I_E$ ), dc emitter current ( $I_E$ ) and unbypassed emitter resistor ( $R_E$ ), one can estimate the resulting level of the AM noise. One can also calculate the maximum allowed value of  $\gamma(\Delta I_E/I_E)$  for a desired AM noise level. In a similar way, one can predict the resulting PM noise due to current fluctuations, but the values of  $C_{cb}$  and  $r_g$  are required.

## VII. MEASUREMENTS OF AM AND PM NOISE INDUCED BY COLLECTOR-BASE VOLTAGE MODULATION IN A CE AMPLIFIER

The collector-base capacitance is a junction capacitance that varies with the dc voltage between the collector and

TABLE I  
AM AND PM NOISE SENSITIVITIES TO CURRENT NOISE FOR A 5 MHz CE AMPLIFIER WHEN  $\gamma = \Delta I_E / I_E = 1.9 \times 10^{-5}$ ,  
 $I_E = 25$  mA, AND  $V_{CB} \approx 9$  V.

	A	B	C	D	E	F	G	H
	2N2222A				2N2222A			
	AM	AM	AM	AM	PM		PM	PM theory
	sensitivity	theory	sensitivity	theory	sensitivity	PM theory $R_E = 0 \Omega$	sensitivity	$R_E = 10 \Omega$
	$R_E = 0 \Omega$	$R_E = 0 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$	$R_E = 0 \Omega$	$R_E = 0 \Omega$	$R_E = 10 \Omega$	[dBc/Hz rel
Fourier	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	to $\gamma = 1]$
frequency	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	
100 Hz	-8.5	-10.5	-23.4	-25.1	-16.3	-15.6	-45.6	-43.7
50 Hz	-8.6	-10.5	-23.3	-25.1	-16.2	-15.6	-45.2	-43.7
20 Hz	-8.6	-10.5	-22.9	-25.1	-15.6	-15.6	-45.8	-43.7
10 Hz	-8.6	-10.5	-22.7	-25.1	-16.3	-15.6	-45.4	-43.7
5 Hz	-8.9	-10.5	-23.3	-25.1	-16.3	-15.6	-45.6	-43.7
					microwave			
	transistor				transistor			
	AM	AM	AM	AM	PM		PM	
	sensitivity	theory	sensitivity	theory	sensitivity	PM theory	sensitivity	PM theory
	$R_E = 0 \Omega$	$R_E = 0 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$	$R_E = 0 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$
Fourier	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel
frequency	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$	to $\gamma = 1]$
100 Hz	-8.5	-10.5	-23.1	-25.1	-43.2	-37.4	limited by	-65.5
50 Hz	-8.5	-10.5	-23.4	-25.1	-42.6	-37.4	system	-65.5
20 Hz	-8.5	-10.5	-23.3	-25.1	-42.1	-37.4	floor	-65.5
10 Hz	-8.5	-10.5	-22.9	-25.1	-42.4	-37.4	< -48.6	-65.5
5 Hz	-8.6	-10.5	-22.5	-25.1	-41.8	-37.4		-65.5

base terminals ( $V_{CB}$ ) according to (37). (In this equation,  $K_0$  is a constant that includes the permittivity of the material and the doping concentration,  $V_{bi}$  is the built-in potential of the p-n junction, and  $n$  is a parameter that depends on the doping profile of the junction.) Collector-base voltage modulation will therefore modulate  $C_{bc}$ :

$$C_{bc} = \frac{K_0}{(V_{bi} + V_{CB})^n}. \quad (37)$$

Fig. 4 shows the relative phase shift across a 20 MHz CE amplifier (shown in Fig. 3) as a function of collector-base voltage ( $V_{CB}$ ) when the active element was a 2N2222A transistor. This graph shows the dependence of  $C_{bc}$  on  $V_{CB}$ , and can be used to obtain the exponent  $n$  in the phase equation of the amplifier (38). The optimum fit, shown as a dotted line in Fig. 4, occurred when  $n \cong 1$ . Fig. 4 shows that the measured phase shift follows (38) fairly well for  $V_{BC}$  from 3 to 24 V.

$$\text{phase} = \vartheta = K_1\omega + K_2\omega C_{bc} = K_1\omega + \frac{K_3\omega}{(V_{bi} + V_{CB})^n}. \quad (38)$$

The PM sensitivity to voltage fluctuations can be obtained by squaring the partial derivative of the phase with respect to  $V_{CB}$ :

$$\left( \frac{\partial \vartheta}{\partial V_{CB}} \right)^2 = \left( \frac{nK_3\omega}{(V_{bi} + V_{CB})^{n+1}} \right)^2. \quad (39)$$

To measure the effect of  $\Delta V_{CB}$  on the AM and PM noise, voltage noise was added to the collector terminal of

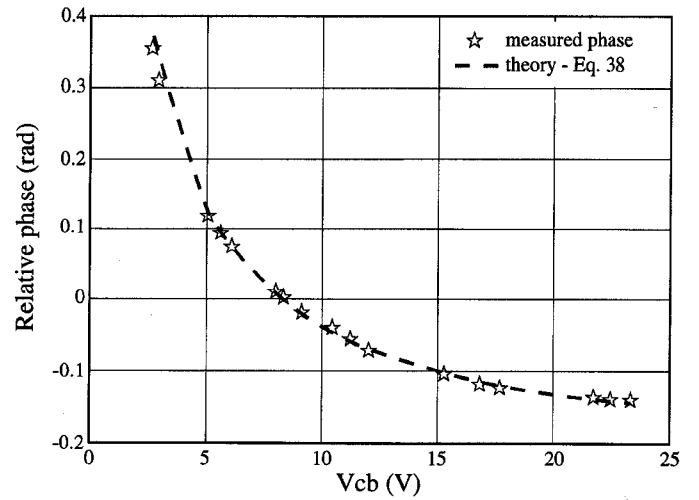


Fig. 4. Relative phase shift of a 20 MHz CE amplifier as a function of  $V_{CB}$ .

TABLE II  
TRANSISTOR PARAMETERS FOR TABLE I.

	2N2222A	Microwave transistor
$f_T$	300 MHz	8 GHz
$C_{be}$	25 pF	1.6 pF
$c_{bc}(V_{CB} = 9V)$	8 pF	.65pF
$\beta$	$75 < \beta < 375$	$50 < \beta < 300$
F	$\leq 4$ dB	1.5 dB

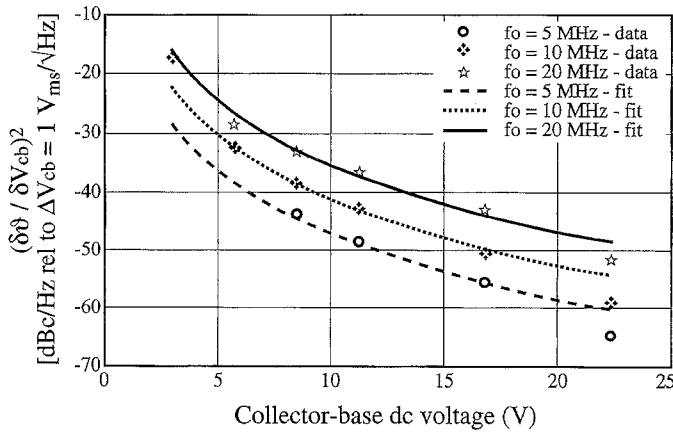


Fig. 5. PM noise sensitivity to  $\Delta V_{CB}$  in a CE amplifier.

the CE amplifier. Filtering on the base reduced any direct current modulation to an insignificant level. Fig. 5 shows the PM noise sensitivity to  $V_{CB}$  fluctuations measured with a 2N2222A transistor in the CE circuit (for  $R_E = 30 \Omega$ ) as a function of  $V_{CB}$  at 5, 10, and 20 MHz carrier frequencies. The dotted lines are predictions on how the PM should vary with  $V_{CB}$  based on (39) for  $n = 1$ . The form of the PM noise follows closely the predictions from 6 to 17 V and scales as  $\omega^2$  as expected.

The change of PM sensitivity to  $V_{CB}$  fluctuations was also measured for different values of  $R_E$  (for a  $V_{CB} = 8$  V). Table III shows the PM sensitivity measurements and predicted values (from (35)) as a function of Fourier frequency offset from a 5 MHz carrier for the CE amplifier in Fig. 3 when voltage noise was injected at the collector terminal. Columns A and D show the measured PM sensitivity to  $V_{CB}$  fluctuations for  $R_E = 0 \Omega$  and  $R_E = 10 \Omega$ , respectively. Columns B and E show the PM sensitivity values predicted by the first term in (35). For the calculations it was assumed that half of the output capacitance in the transistor was due to parasitics and the other half due to the junction capacitance (37).

The measured and predicted values for the PM sensitivity when a 2N2222A was used differ by approximately 4 dB. This is probably due to the approximations made in (35) and the uncertainty in some of the values used ( $r_g, C_{bc}$ ). From theory, the reduction in PM when  $R_E$  is added should be proportional to the reduction in gain. Both the measured and calculated sensitivities show approximately a 12 dB reduction for  $R_E$  changed from 0  $\Omega$  to 10  $\Omega$ . The measured gain reduction was 13.5 dB. When a microwave transistor was used, the PM sensitivity to  $V_{CB}$  fluctuations for  $R_E = 0 \Omega$  was reduced by 33 dB, compared to the predicted reduction of 28 dB. The predicted value was obtained from (35) and (39) when using  $n = 0.55$  and  $C_{bc} = 0.3$  pF. These values were obtained from the  $C_{bc}$  vs.  $V_{CB}$  plot in the specification sheet of the transistor. For  $R_E = 10 \Omega$  the noise was limited by the noise floor of the measurement system. The AM sensitivity to  $V_{CB}$  fluctuations is shown in columns C ( $R_E = 0 \Omega$ ) and F ( $R_E = 10 \Omega$ ) of Table III. The AM sensitivity when

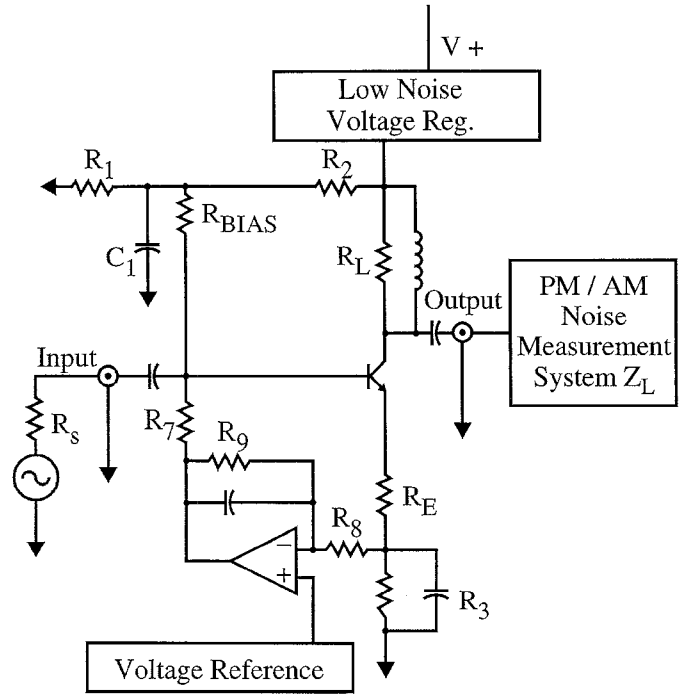


Fig. 6. CE amplifier with active feedback to stabilize the emitter current.

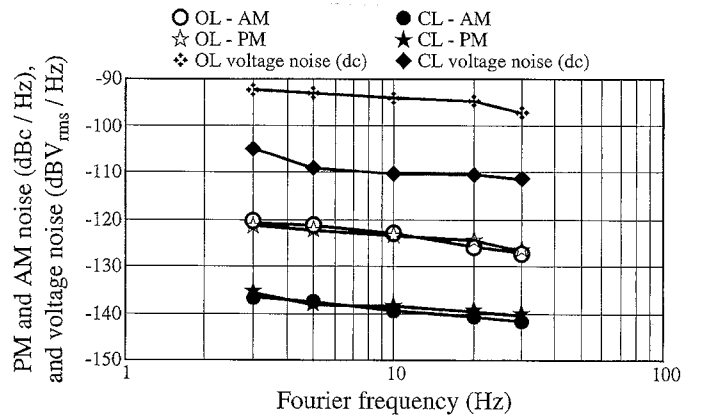


Fig. 7. Voltage noise, AM noise and PM noise of a 5 MHz CE amplifier for open loop (OL) and closed loop (CL) configurations.

using a 2N2222A was reduced by approximately twice the amount of reduction in PM sensitivity when an unbypassed emitter resistor was added ( $R_E = 10 \Omega$ ), as predicted by theory. The PM noise is thus a first order effect in the up-conversion of  $V_{CB}$  fluctuations, while the AM noise is a second order effect.

## VIII. REDUCTION OF AM AND PM NOISE USING ACTIVE FEEDBACK

The AM and PM noise due to current fluctuations can be reduced considerably by adding a servo system that reduces the current noise. Fig. 6 illustrates such a system. A circuit that reads the current at the emitter, amplifies and inverts it, and injects the signal back into the base

TABLE III

AM AND PM NOISE SENSITIVITIES TO COLLECTOR-BASE VOLTAGE FLUCTUATIONS FOR A CE AMPLIFIER WITH  $V_{CB} = 8$  V AND  $I_E = 25$  MA. FOR THESE MEASUREMENTS THE OUTPUT POWER LEVEL WAS ADJUSTED TO BE APPROXIMATELY 6 DBM AND  $\Delta V_{CB} \cong 2.8 \times 10^{-4} V_{rms}/\sqrt{\text{Hz}}$ .

	A	B	C	D	E	F
	2N2222A					
	PM	PM	AM	PM	PM	AM
	sensitivity	theory	sensitivity	sensitivity	theory	sensitivity
	$R_E = 0 \Omega$	$R_E = 0 \Omega$	$R_E = 0 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$	$R_E = 10 \Omega$
	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel	[dBc/Hz rel
Fourier	to $\Delta V_{CB} =$	to $\Delta V_{CB} =$	to $\Delta V_{CB} =$	to $\Delta V_{CB} =$	to $\Delta V_{CB} =$	to $\Delta V_{CB} =$
frequency	$1V_{rms}/\sqrt{\text{Hz}}$	$1V_{rms}/\sqrt{\text{Hz}}$	$1V_{rms}/\sqrt{\text{Hz}}$	$1V_{rms}/\sqrt{\text{Hz}}$	$1V_{rms}/\sqrt{\text{Hz}}$	$1V_{rms}/\sqrt{\text{Hz}}$
100 Hz	-40.7	-36.2	-42.4	-52.3	-48.6	-63
50 Hz	-40.7	-36.2	-42.6	-52.4	-48.6	-62.8
20 Hz	-40.6	-36.2	-42.8	-52.4	-48.6	limited by
10 Hz	-40.6	-36.2	-43.1	-52.4	-48.6	system
5 Hz	-40.6	-36.2	-43.1	-52.4	-48.6	noise floor
	microwave					
	transistor					
100 Hz	-73.6	-63.9	limited by	limited by	-76.3	limited by
50 Hz	-74.3	-63.9	system	system	-76.3	system
20 Hz	-73.2	-63.9	noise floor	noise floor	-76.3	noise floor
10 Hz	limited by	-63.9	< -63	< -77	-76.3	< -69
5 Hz	sys. floor	-63.9			-76.3	

was added to the CE amplifier. Noise was added to the base of the transistor causing fluctuations in the emitter current. Emitter current fluctuations are up-converted to AM and PM noise, and voltage fluctuations at the base also up-convert to PM noise. The servo reduces both the current fluctuations and the voltage fluctuations at the base; therefore, it should also reduce the AM and PM noise.

Fig. 7 shows the measured emitter voltage noise (proportional to current noise), and the AM and PM noise of the CE amplifier for open loop and close loop configurations. When the loop was closed, the voltage noise, the AM noise, and the PM noise were reduced by the amount of the gain of the servo amplifier, as expected.

## IX. CONCLUSION

We have presented a theory that explains the up-conversion of baseband noise into AM and PM noise about a carrier in BJT amplifiers. According to this theory, AM and PM noise equations were developed for CE amplifiers, but the analysis can also be applied to any BJT amplifier configuration. By injecting current noise and voltage noise in a CE amplifier, we were able to verify to a few dB the leading term in both AM and PM noise due to baseband current modulation and collector-base voltage modulation. Based on this study, we were able to confirm the functional dependence of AM and PM noise on transistor parameters, circuit parameters, and carrier frequency given by the derived noise equations. The developed theory lays the groundwork for a comprehensive theory of AM and PM noise in BJT amplifiers. In addition, the derived

noise equations that describe the origin of AM and PM noise provide a basis for the design of low noise amplifiers.

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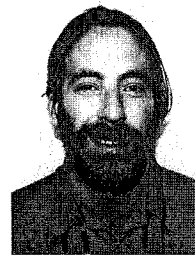
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